

1.2A Review of Solving Linear Equations

A. Equations vs. Non-equations

Equations have equal signs (=) in them. Solutions are typically numbers.

Non-equations do not have equal signs in them. Answers to these problems are typically ugly expressions with variables

Notice the difference:

① Combine $\frac{5}{x-3} + \frac{2}{x+1}$

[non-equation: answer is ugly expression found by rewriting each fraction with common denominator; See MTH1825 Sec 6.2]

② Solve $\frac{5}{x-3} + \frac{2}{x-1} = 7$

[equation: method here is completely different
This is a rational equation solved by
MTH1825 Sec 6.5, reviewed in 1.2C]

B. Linear Equations

Equations with no variables in denominators and no powers on the variables

i.e. $ax+b=c$ [where $a \neq 0$]

C. Solving Linear Equations

1. Simplify each side: (A) Clear Parentheses
(B) Clear Fractions
(C) Combine Like Terms

2. Isolate the variable to one side

3. Solve

4. Check (plug value back in to original equation, and see if true)

Note! In MTH103, we put the number answer (by itself)
in a set $\{\}$

i.e. if $x=3$, we write $\{3\}$

D. Examples

Example 1: Solve $3-5(x-2)=2x+1$ for x

Solution

Step 1: (A) Clear Parentheses: $3-5x+10=2x+1$
(B) Clear Fractions ✓
(C) Combine Like Terms: $13-5x=2x+1$

Step 2: $13-7x=1$
 $-7x=-12$

Step 3: $x = \frac{-12}{-7} = \frac{12}{7}$

Step 4: Check: $3-5\left(\frac{12}{7}-2\right) \stackrel{?}{=} 2\left(\frac{12}{7}\right)+1$
 $3-5\left(-\frac{2}{7}\right) \stackrel{?}{=} \frac{24}{7}+1$
 $3+\frac{10}{7} \stackrel{?}{=} \frac{31}{7} \checkmark$

Ans: $\left\{\frac{12}{7}\right\}$

Example 2: Solve $\frac{x+1}{4} + \frac{3-x}{3} = \frac{5}{6}$ for x

Solution

Step 1: ④ Clear Parentheses ✓

③ Clear Fractions:

$$12 \left[\frac{x+1}{4} + \frac{3-x}{3} \right] = 12 \left[\frac{5}{6} \right]$$

$$3(x+1) + 4(3-x) = 10$$

using diagonal
cancellation
at each term

$$3x+3+12-4x=10$$

⑤ Combine Like Terms:

$$-x+15=10$$

Step 2: $-x = -5$

Step 3: $x = 5$

Step 4: Check: $\frac{5+1}{4} + \frac{3-5}{3} \stackrel{?}{=} \frac{5}{6}$

$$\frac{6}{4} + \frac{-2}{3} \stackrel{?}{=} \frac{5}{6}$$

$$\frac{18}{12} - \frac{8}{12} \stackrel{?}{=} \frac{5}{6} \quad \checkmark$$

Note: The check is not mandatory, but it is a good idea

Ans: $\{5\}$

E. Special Cases

When solving an equation, sometimes "x" disappears.

1. If the statement is true (e.g. $0=0$), the equation is true for all real numbers. This is called an identity and the answer is written as $(-\infty, \infty)$.
2. If the statement is false (e.g. $1=2$), the equation has no solutions. This is called an inconsistent equation and the answer is written as $\emptyset$ $\leftarrow$ symbol for empty set

Sidebar: "Normal equations" that don't behave like these are called conditional equations.

Example 1: Solve $3(5-x) = 2x+4-5x$ for x

Solution

$$15 - 3x = -3x + 4$$

$$15 = 4$$

(adding $3x$ to each side)

Ans: $\boxed{\emptyset}$

Example 2: Solve $3(2x+7)-1 = 6x+20$ for x

Solution

$$6x + 21 - 1 = 6x + 20$$

$$6x + 20 = 6x + 20$$

$$20 = 20$$

Ans: $\boxed{(-\infty, \infty)}$