MATH 425 TEST II: EXTRA CREDIT (OPTIONAL) DUE: APRIL 20, 2011

Name:____

Instructions:

- You should work alone **(i.e. you may not consult other students)**, but you may consult the course instructor, the course texts and your lecture notes.
- Due in class on April 20, 2011.
- Unless indicated otherwise, you may use any theorem proved in the lecture. When applying a theorem in a problem be sure to indicate clearly that you are doing so, and verify that all necessary hypotheses are satisfied.
- Explain your answers using **complete sentences**.

Sign your name on the line below to indicate that you have read the instructions and have not received any unauthorized assistance in the completion of this assignment.

Signature:____

Problem 1 (10 points)	
Problem 2 (15 points)	
Problem 3 (5 points)	
Problem 4 (5 points)	
Problem 5 (5 points)	
Total (40 points)	

1. Evaluate the following line integrals.

   (a) $\int_C \bar{z}\, dz$, where C is the straight-line segment connecting 0 to $2 + 2i$.

   (b) $\int_C z \cos^2 z\, dz$, where C is the boundary of the triangle with vertices 0, i, and $1 + i$ traversed once around in the counter-clockwise direction.

2. (a) State the Cauchy Integral Formula. In your statement, use **complete sentences** to explain all notation and any assumptions which are necessary for the Cauchy Integral Formula to be true.

(b) Let $f(z) = \frac{z^2}{z^2+2z+2}$. Evaluate the line integral $\int_C f(z)\,dz$ where C is the smooth curve defined by:

i. $z(t) = i + 2e^{it} \quad t \in [0, 2\pi]$

ii. $z(t) = -1 - i + e^{-i2t} \quad t \in [0, 2\pi]$

3. Suppose that the power series $\sum_{k=0}^{\infty} a_k(z-4)^k$ satisfies

$$\sum_{k=0}^{\infty} a_k(z-4)^k = \frac{\cos z}{z^2+9}$$

for all z in some open set containing $z = 4$. Find the radius of convergence of this power series, and explain why you know your answer is correct.

4. Suppose that f is an entire function satisfying

$$|f(z)| \leq |z|^5$$

for all $z \in \mathbb{C}$. Use the ML-formula and the formula

$$f^{(k)}(0) = \frac{k!}{2\pi i} \int_C \frac{f(z)}{z^{k+1}}\, dz$$

where C is a circle centered at 0 traversed once in the counter-clockwise direction to show that the k-th derivative $f^{(k)}(z)$ satisfies $f^{(k)}(0) = 0$ for all $k \geq 6$. (You are not permitted to use Theorem 5.11 in this problem)

5. Let $f : \mathbb{C} \to \mathbb{C}$ be a continuous (but not necessarily analytic) function, and for $z \in \mathbb{C}$ let C_z be the smooth curve defined by

$$z(t) = tz \quad t \in [0, 1],$$

so that C_z is the straight line segment connecting 0 and z. Define a function $F : \mathbb{C} \to \mathbb{C}$ by

$$F(z) = \int_{C_z} f(w)\, dw.$$

Prove that $\lim_{z \to 0} \frac{F(z)}{z} = f(0)$.