Math 425, Homework #4
Due: April 25, 2011

Instructions:

- Write in compete sentences, organized into paragraphs. Remember you should be writing your solutions so that someone who knows neither the question nor answer could read them and understand what you are proving or computing.
- Leave plenty of room in between problems.
- Write only on the front side of each sheet of paper.
- Staple!
- Write on your assignment the names of any persons or sources consulted during its completion (other than the course text or instructor).

(1) (8.9) Define a function f analytic in the plane minus the non-positive real axis and such that $f(x) = x^x$ on the positive real axis. Find $f(i)$, and $f(-i)$. Show that $f(\bar{z}) = \overline{f(z)}$ for all z in the domain.

(2) Prove that the function $f(z) = \frac{\cos z - 1}{z^2}$ has a removable singularity at $z = 0$.

(3) (9.10) Find the principal part of the Laurent expansion of

$$f(z) = \frac{1}{(z^2+1)^2}$$

about the point $z = i$.

(4) (9.12) Find the Laurent expansion of $f(z) = \frac{1}{z(z-1)(z-2)}$ (in powers of z) for

(a) $0 < |z| < 1$
(b) $1 < |z| < 2$
(c) $|z| > 2$.