Math 425, Homework #2
Due: March 18, 2011

Instructions:

- Write in compete sentences, organized into paragraphs. Remember you should be writing your solutions so that someone who knows neither the question nor answer could read them and understand what you are proving or computing.
- Leave plenty of room in between problems.
- Write only on the front side of each sheet of paper.
- Staple!
- Write on your assignment the names of any persons or sources consulted during its completion (other than the course text or instructor).

(1) Prove that for z, $w \in \mathbb{C}$, $e^z = e^w$ if and only if there exists a $k \in \mathbb{Z}$ so that $z = w + i2\pi k$.

(2) (3.19) Find all solutions to the equation

$$e^{(e^z)} = 1.$$

(3) (a) Show that for complex numbers z, w that

$$\cos z = \cos w \text{ and } \sin z = \sin w$$

if and only if $z = w + 2k\pi$ for some $k \in \mathbb{Z}$.

(b) Show that

$$\cos(z + w) = \cos z \cos w - \sin z \sin w$$

for any z, $w \in \mathbb{C}$.

(4) Compute $\int_C z^2 \bar{z}\, dz$ where C is the curve defined by

$$z(t) = t + it^2 \text{ for } t \in [0, 1].$$

(5) Let u, $v : \mathbb{C} \to \mathbb{R}$ be continuous functions and let x, $y : [a, b] \to U \subset \mathbb{R}$ be differentiable functions. Define $f(z) = u(z) + iv(z)$ and define C to be the curve $z(t) = x(t) + iy(t)$ for $t \in [a, b]$. Find formulas for the real and imaginary parts of $\int_C f(z)\, dz$ in terms of integrals of expressions involving u, v, x, and y.