Use applicable theorems to compute

$$\int_C \frac{2z}{z^2+1}\,dz$$

where C is the curve given by:

(1) $z(t) = i + e^{it}$ for $t \in [0, 2\pi]$

(2) $z(t) = -i + e^{it}$ for $t \in [0, 2\pi]$

(3) $z(t) = -1 + e^{it}$ for $t \in [0, 2\pi]$.

(4) $z(t) = 1 + i + 2e^{it}$ for $t \in [0, 2\pi]$

(5) $z(t) = 1 + i + 2e^{-it}$ for $t \in [0, 4\pi]$

(6) $z(t) = 3e^{it}$ for $t \in [0, 2\pi]$ (Hint: use a partial fraction decomposition of $\frac{z}{z^2+1}$.)

(Answers on the next page).

Answers: For each part it is convenient to rewrite the integral as $\int_C \frac{2z}{(z+i)(z-i)}\,dz$.

(1) $2\pi i$ (Apply Cauchy Integral Formula with $a = i$ and $f(z) = \frac{2z}{z+i}$.)

(2) $2\pi i$ (Apply CIF with $a = -i$ and $f(z) = \frac{2z}{z-i}$.)

(3) 0 (Apply the closed curve theorem to $f(z) = \frac{2z}{z^2+1}$.)

(4) $2\pi i$ (Apply CIF with $a = i$ and $f(z) = \frac{2z}{z+i}$.)

(5) $-4\pi i$ (Use problem (4) with Proposition 4.7.)

(6) $4\pi i$ (Rewrite $\int_C \frac{2z}{(z+i)(z-i)}\,dz = \int_C \frac{1}{z+i}\,dz + \int_C \frac{1}{z-i}\,dz$. Apply CIF with $f(z) = 1$ and $a = -i$ to the first integral on the right and apply CIF with $f(z) = 1$ and $a = i$ to the second integral on the right.)