

Section 2: Algebraic Calculations

Maple is a "C.A.S", i.e., a Computer Algebra System. This means that Maple knows every rule of algebra that you know. As you progress through calculus, differential equations and linear algebra you will find that Maple also has the essential operations from those subjects built into its large command set.

In this section you will learn how to enter an algebraic expression and substitute values in for the variables. Then you will learn the commands that allow you to expand, factor and simplify expressions.

```
> restart;  
>
```

The *eval* command

Example 1

For our first example let's start with the expression $3x^2 + 8$ and assign it the name *W*.

```
> W := 3*x^2+8;  
>
```

Suppose now that you want to evaluate the expression $3x^2 + 8$ where *x* has the value 4. The quickest way to do this is to use Maple's **eval** command. Here's what it looks like:

```
> eval(3*x^2+8,x=4);  
>
```

Alternatively, you can apply the **eval** command to *W*.

```
> eval(W,x=4);  
>
```

Example 2

The **eval** command works equally well with symbolic values:

To replace *x* by $5 + 2u$ in the expression $3x^2 + 8$ execute the following line. In this case we label the result *M*.

```
> W := 3*x^2+8;  
>  
> M := eval(W,x=5+2*u);  
>
```

And now to get Maple to "multiply out" this expression we use the **expand** command.

```
> expand(M);  
>
```

Example 3

The **eval** command is very versatile. You can use it to evaluate expressions involving more than one variable. Here we replace *x* by 7 and *y* by 12 in the expression $U = \frac{2x^2}{5} + 3y$.

```
> U := (2/5)*x^2+3*y;
```

```
>
```

Since U has two variables, the evaluating point then needs to be in a list, in square brackets [] separated by commas. Note $x=7$, $y=12$ are enclosed by a square bracket [].

```
> q := eval(U, [x=7, y=12]);
```

```
>
```

As a floating-point (decimal) number, we have

```
> evalf(q);
```

```
>
```

Example 4

You can also use the **eval** command to substitute a value into an equation. This is the sort of thing you might want to do to test whether a particular value "satisfies" the equation. In the next few lines we substitute different values into the equation $x^3 - 5x^2 + 7x - 12 = 0$. Are any of these values a solution to the equation?

Note we use " := " to assign the name and just "=" for the equation itself.

```
> eqn := x^3-5*x^2+7*x-12=0;
```

```
>
```

To see if 3 is a solution for eqn, execute:

```
> eval(eqn, x=3);
```

```
>
```

Similarly, to see if 4 is a solution for eqn, execute:

```
> eval(eqn, x=4);
```

```
>
```

As you can see, since -9 is not equal to 0, 3 is not a solution to $x^3 - 5x^2 + 7x - 12 = 0$, and $0 = 0$ indicates that 4 is in fact a solution. Again, 5 is not a solution as $x^3 - 5x^2 + 7x - 12 = 23$ when $x = 5$, which the following shows.

```
> eval(eqn, x=5);
```

```
>
```

Exercise 2.1

Assign the name k to the expression $x^2 + 4x - 3$. Then assign the name M to the expression $k^2 - 9$.

Finally have Maple calculate $3M + 6$.

Note: to get Maple to multiply the expression out use the **expand** command. That is enter: **expand(3*M+6)**; You will learn more about the expand command in the next subsection.

▼ *Student Workspace 2.1*

```
[>  
[>  
[>  
[>  
[>  
[>
```

▼ *Answer 2.1*

```
[> k := x^2+4*x-3;  
[>  
[> M := k^2-9;  
[>  
[> 3*M+6;  
[>  
[> expand(3*M+6);  
[>
```

▼ Exercise 2.2

Expand $(1 + x)^4$ using the **expand** command.

▼ *Student Workspace 2.2*

```
[>  
[>  
[>  
[>
```

▼ *Answer 2.2*

```
[> w := (1+x)^4;  
[>  
[> expand(w);  
[>  
or we can do this all in one step with:  
[> expand((1+x)^4);  
[>
```

▼ Exercise 2.3

Let $P = ax^3 + bx^2 + cx + d$. Find P if $x = 0.01$, $a = -\frac{1}{5}$, $b = \frac{2}{5}$, $c = 0$, and $d = \frac{13}{15}$.

▼ *Student Workspace 2.3*

```
[>  
[>  
[>  
[>
```

▼ *Answer 2.3*

```
[> P := a*x^3+b*x^2+c*x+d;  
[>  
[> eval(P, [x=0.01, a=-1/5, b=2/5, c=0, d=13/15]);
```



```
> expand(cos(2*x));
```

```
>
```

Try expanding the sine and cosine of some other integer multiples of x . For example: $\sin(3x)$, $\cos(6x)$, etc.

```
>
```

Example 3

Here is a final example. Have Maple multiply out the expression:

$$x^{\left(\frac{1}{2}\right)} \left(x^{\left(\frac{3}{2}\right)} + x^{\left(-\frac{1}{2}\right)} \right)$$

```
> h := x^(1/2)*(x^(3/2)+x^(-1/2));
```

```
>
```

```
> expand(h);
```

```
>
```

Exercise 2.5

Expand $(x + 1)^n$ for $n=2,3$ and 4 .

Student Workspace 2.5

```
>
```

```
>
```

```
>
```

```
>
```

```
>
```

```
>
```

```
>
```

```
>
```

```
>
```

Answer 2.5

```
> expand((x+1)^2);
```

```
>
```

```
> expand((x+1)^3);
```

```
>
```

```
> expand((x+1)^4);
```

```
>
```

The factor command

Example 1

Factor the expression: $3x^2 - 10x - 8$

```
> w := 3*x^2-10*x-8;
```

```
>
```

```
> factor(w);
```

```
>
```

Or you can do it all on one line:

```
> factor(3*x^2-10*x-8);
```

L L >

▼ **Example 2**

First enter in the expression $2(x - 2)(2x^2 + 5x + 2)(x + 4)$.

```
> H := 2*(x-2)*(2*x^2+5*x+2)*(x+4);
```

Expand H using the **expand** command.

```
> ans := expand(H);
```

Then apply the **factor** command to the result.

```
> factor(ans);
```

Can you explain why the final result looks different than the original expression ?

▼ **Example 3**

Maple can factor expressions with more than one variable. For example, the expression $x^2y + 2xy + y$, that is,

```
> h := x^2*y+2*x*y+y;
```

factors to

```
> factor(h);
```

▼ **Example 4**

If Maple can't factor an expression using rational numbers (i.e. integers and fractions) then it returns the input unchanged.

```
> factor(3*x^2-10*x-9);
```

▼ **Example 5**

The **factor** command is not limited to polynomials. It can be used to factor other forms.

Factor $\sin^2 x - \cos^2 x$.

```
> factor((sin(x))^2-(cos(x))^2);
```

▼ **Example 6**

If the **factor** command is used with a rational expression such as

```
> A := (x^3-7*x^2+15*x-9)/(x^2+4*x+4);
```

the numerator and denominator are each factored, as we see from

```
> factor(A);
```

```
>
```

The common factors in the expression

```
> B := (x^3-7*x^2+15*x-9)/(x^2-4*x+3);
```

```
>
```

are cancelled to simplify the expression, as we see from

```
> factor(B);
```

```
>
```

The next example allows you to see the factored form without cancellation.

```
>
```

Example 7

Maple's **numer** and **denom** commands allow you to isolate either the numerator or denominator of a fraction.

For example, consider $\frac{x^3 - 7x^2 + 15x - 9}{x^2 - 4x + 3}$.

```
> B := (x^3-7*x^2+15*x-9)/(x^2-4*x+3);
```

```
>
```

Here we use **numer** and **denom** commands to examine the factors of the numerator and denominator separately (i.e. before cancellation of common factors).

```
> factor(numer(B));  
factor(denom(B));
```

```
>
```

Exercise 2.6

Factor the expression $3x^4 - 2x^3 + 22x^2 - 18x - 45$.

Student Workspace 2.6

```
>  
>  
>  
>
```

Answer 2.6

```
> factor(3*x^4-2*x^3+22*x^2-18*x-45);
```

```
>
```

Exercise 2.7

Factor the expression $\sqrt{x} - x^{\left(\frac{3}{2}\right)}$ and then use the **expand** command to check the result.

Student Workspace 2.7

```
>  
>  
>  
>  
>
```

Answer 2.7

```
> ww := x^(1/2)-x^(3/2);  
>  
> factor(ww);  
>  
> expand(%);  
>
```

The *simplify* command

Example 1

Maple can apply identities to simplify many lengthy mathematical expressions, such as trigonometric expressions. For example, consider the expression

$$\cos(x)^5 + \sin(x)^4 + 2 \cos(x)^2 - 2 \sin(x)^2 - \cos(2x)$$

that is,

```
> V := cos(x)^5+sin(x)^4+2*cos(x)^2-2*sin(x)^2-cos(2*x);  
>
```

which Maple simplifies to

```
> simplify(V);  
>
```

Example 2

Trigonometric expressions with arguments in multiples of some angle will be simplified to trig functions in the single angle if possible:

```
> simplify(sin(5*t)+sin(3*t));  
>
```

Example 3

The **simplify** command can be used to add rational expressions.

Rewrite the sum $\frac{1}{x+1} + \frac{x}{x-1}$, that is,

```
> M := (1/(x+1))+(x/(x-1));  
>
```

as a single fraction via

```
> simplify(M);  
>
```

▼ ***Exercise 2.8***

Simplify the expression $\frac{7}{x+2} + \frac{3x}{(x+2)^2}$

▼ ***Student Workspace 2.8***

```
[>  
[>  
[>
```

▼ ***Answer 2.8***

```
[> simplify(7/(x+2)+(3*x)/(x+2)^2);  
[>
```

▼ ***Exercise 2.9***

How does Maple simplify $\sin(3t) - \sin(7t)$? Whether or not this "simplified" form is of use to you will depend on what you plan to do with it.

▼ ***Student Workspace 2.9***

```
[>  
[>  
[>  
[>
```

▼ ***Answer 2.9***

```
[> h := sin(3*t)-sin(7*t);  
[>  
[> simplify(h);  
[>
```