Complex Arithmetic (mini-tutorial)

Maple represents i, the "square root of -1" by the symbol "I". It treats numerical expressions like 3+4*I just like any other numbers. For example:

```
> s:=sqrt(I);
```

Note that Maple has computed the Principal value!

To get both square roots we try:

```
> solve(z^2=I);
```

Exercises for you:
(a) Find the Principal value of the square root of 1+i. Then find both square roots.
(b) Find all cube roots of 1+i and check your answers with paper and pencil.

Real and imaginary parts are computed using the commands "Re" and "Im":

```
> Re(s); Im(s);
```

Maple does complex arithmetic using the same commands as for real arithmetic; you might check by hand that the calculations below are correct:

```
> z_1:=3+4*I; z_2:=1+2*I;
> z_1+z_2; z_1*z_2;z_1/z_2;
```

You find the absolute value and (principal value of the) argument of a complex number by using the commands "abs" and "argument":

```
> abs(z_1); argument(z_1);
```

Exercises for you:
(a) How do you get Maple to approximate this last answer by a decimal number?
(b) Find the absolute values and arguments of the cube roots of 1+i.

Complex Functions (mini-tutorial)

Let's first try defining the complex variable z by $z = x + iy$:

```
> z:=x+I*y;
```

What's the real and imaginary part?

```
> Re(z); Im(z);
```

The problem here is that Maple doesn't yet know that x and y are supposed to be real! We inform it of this by using the "assume" command:

```
> assume(x,real); assume(y,real);
```

Now the Re and Im commands work as we expect they should:

```
> Re(z); Im(z);
```

The tilde ("~") in the output after x and y indicates that an assumption has been placed on these variables. When using the variables in commands, however, just type x and y; don't put in the tilde yourself! For example, to compute e^z and its real and imaginary parts:

```
> expofz:=exp(z);
> Re(expofz); Im(expofz);
```

Even more useful; we can use Maple to compute real and imaginary parts of more complicated functions. Remember, we've defined z to be $x + iy$, where x and y have been "assumed" real:

```
> f:=sin(z^2+1);
> Re(f);Im(f);
```

Exercise for you: check by hand that these answers are correct!

You differentiate complex functions just like you do for real ones. The idea is to hide from Maple the fact that z is a complex variable by invoking the "restart" command:

```
> restart;
```

Let's compute some derivatives of $\sin(z^2+1)$:

```
> f:=sin(z^2+1);
> d1f:=diff(f,z);
> d2f:=diff(d1f,z);
```

Or, you can do the second derivative directly:

```
> d2f2:=diff(f,z,z);
```

For , say, the tenth derivative, instead of typing ten repetitions of "z," one can use:

```
> diff(f,z$10);
>
```

Homework Problems (extra credit, 5 points each)

Instructions: You may hand in solutions to these problems either on paper or online. Whichever way you choose, *be sure to include enough comments so that I can follow what you are doing.*

1. Determine the real and imaginary parts of the second derivative of *f(z)*=exp*(z^2)*. Success here will depend on "assuming" x and y to be real *at the appropriate moment!* Check at least one of these results with pencil and paper.

```
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>
```

2. Show that the real and imaginary parts of the second derivative of exp*(z^2)* are harmonic (i.e. satisfiy Laplace's equation).

```
>
>
>
>
```

3. Determine all the zeros of the polynomial $p(z) = z^7 + 3z + 1$. Which one has the largest magnitude? The largest argument? Which is real? (Best to "restart" here to clear the definition of "*z*").

```
>
>
>
>
>
>
```

4. Plot the zeros of the polynomial in Exercise 3 above. Use "complexplot" which is part of the "plots" package you load with the command "with(plots):" use "style=point" and "symbol=circle".

```
>
>
```

The non-real solutions appear to be happening in complex conjugate pairs. Can you explain why this is true? State and prove a theorem that generalizes this phenomenon.

5. The solutions to the equation $p(z) = 0$ are actually solutions to the system of real polynomial equations in x and y: $u(x,y) = \mathrm{Re}(p(z)) = 0$, $v(x,y) = \mathrm{Im}(p(z)) = 0$.

(a) Use Maple to find these two polynomials as functions of x and y (don't forget to define $z = x + \mathrm{i}y$ with x and y real, as in Problem #1 above).

(b) Use Maple's "implicitplot" command (part of the "plots" package) to graph the curves $u = 0$ and $v = 0$ in different colors on the same set of axes.

(Assign names like "plot1" and "plot2" to the plots you made above, and superimpose them using the "display" command. Choose appropriate ranges for x and y so the intersections are shown clearly.)

>
>
>
>
>

6. Superimpose the plot you made in #4 above on the plot you made in #5 to show that the solutions you obtain in both cases are the same.

>
>
>