

Assignment #10*Due date-TBA*

1. Exercise 2.4.1
2. Show that the following three statements about a sequence (a_n) of real numbers are equivalent (in the sense that each implies the others):

(a) $\lim_{n \rightarrow \infty} a_n = a.$

(b) $\lim_{n \rightarrow \infty} (a_n - a) = 0.$

(c) $\lim_{n \rightarrow \infty} |a_n - a| = 0.$

Suggestion: Write down the “ $\varepsilon - N$ ” definition of limit for each. Notice that you’ve written down the same thing three times!

3. Prove that $\lim_{n \rightarrow \infty} a_n = a$ if and only if $\lim_{n \rightarrow \infty} a_{n+1} = a.$

4. Prove that $\lim_{n \rightarrow \infty} a_n = a \implies \lim_{n \rightarrow \infty} a_{2n} = a.$

Is the converse true? If so, prove it. If not, provide an example.

5. *Geometric series.* Suppose x is a real number not equal to 1.

- (a) Prove by induction that

$$1 + x + x^2 + x^3 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}. \quad (1)$$

- (b) Now give a second proof by multiplying the left-hand side of (1) by $1 - x$ and observing that the sum you get “telescopes”.
- (c) Use (1) to help you determine for which real x the *Geometric Series*

$$1 + x + x^2 + x^3 + \cdots$$

converges, and to find its sum when it does converge.