

Assignment #1

Due Friday, January 13, before start of class

1. Where does the argument proving “ $\sqrt{2}$ is irrational” break down when we try to use it to prove the obviously false statement “ $\sqrt{4}$ is irrational?”

2. To prove the irrationality of $\sqrt{2}$ we needed the fact that the square of any odd integer is odd. Prove that, more generally, *the product of any pair of odd integers is odd.*

Suggestion: Use the fact that an integer is odd iff (if and only if) it has the form $2n + 1$ for some $n \in \mathbb{Z}$.

3. Call an integer a “triple” if it is divisible by 3, i.e., if it has the form $3n$ for some $n \in \mathbb{Z}$ (here we use the convention that *in a definition*, “if” means “if and only if”). Show that *the product of any pair of integers that are non-triples is again a non-triple.*

Suggestion: Use the fact that an integer is a non-triple iff it has the form $3n + j$ where $n \in \mathbb{Z}$ and $j = 1$ or $j = 2$.

4. Show that $\sqrt{3}$ is irrational.