PROBLEM (7) FROM SECTION 3.2

DAVID SEAL

Example 1. *On* $\mathbb{R}_2[t]$, *let* $(T_1\mathbf{p})(t) = \mathbf{p}(t-1)$. *Find the matrix of* T_1.

Note that $T_1 : \mathbb{R}_2[t] \to \mathbb{R}_2[t]$ maps functions to new functions. Functions are defined by how they act on their arguments, in this case t.

The standard basis for $\mathbb{R}_2[t]$ is the set, $\{1, t, t^2\}$. That is, as functions, we can write $\mathbf{b}_1(t) = 1$, $\mathbf{b}_2(t) = t$, and $\mathbf{b}_3(t) = t^2$.

If we evaluate these functions at $t = t - 1$, we see how T_1 operates on them:

$$\mathbf{b}_1(t-1) = 1, \quad \mathbf{b}_2(t-1) = t-1, \quad \text{and} \quad \mathbf{b}_3(t-1) = (t-1)^2.$$

Therefore,

$$(T_1\mathbf{b}_1)(t) = 1, \quad (T_1\mathbf{b}_2)(t) = t-1, \quad \text{and} \quad (T_1\mathbf{b}_3)(t) = (t-1)^2.$$

If we write each of these in the standard basis, we have

$$[T_1\mathbf{b}_1]_{\mathcal{E}} = (1,0,0)^T, \quad [T_1\mathbf{b}_2]_{\mathcal{E}} = (-1,1,0)^T, \quad [T_1\mathbf{b}_3]_{\mathcal{E}} = (1,-2,1)^T.$$

The matrix for the linear transformation is constructed by stacking each of these vectors next to each other:

$$[T_1]_{\mathcal{E}} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}.$$

Date: July 13, 2013.