HOMEWORK SET 6

MATH 415, SUMMER 2013

1. Reading

Read chapter 6, sections 3-7 and 9.

2. Problems

All of the following problems are from Sadun's text.

(1) Section 6.3, problems 1–4, 6-7.
(2) Section 6.4, problems 1–4.
(3) Section 6.5, problems 5–8, 10–12,
(4) Section 6.6, problems 1,2
(5) Section 6.7, problems 2–4, 8, 10.
(6) Section 6.9, problems 1,2, 4, 5.

Here are some additional exercises to think about.

(1) Define the following map, $< \cdot, \cdot >: V \times V \to \mathbb{R}$ by

$$\int_{-1}^{1} f(t)g(t)\sqrt{1-t^2}\,dt,$$

where $V = \mathbb{R}[t]$. Show that this is a *bilinear form*. Show that this is *symmetric*, and *positive*, and therefore conclude that this is an inner product.

(2) For this problem only, we will use x in place of t as our independent variable. Consider a function ϕ expanded in its Fourier basis:

$$\phi(x) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right),$$

where the a_n's are unknown quantities.
   (a) Show that $\phi(0) = \phi(1) = 0$.
   (b) Suppose $\phi''(x) = f(x)$, where

$$f(x) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{L}\right),$$

is a known function. Write down a formula for the a_n in terms of the c_n. *Hint:* equate coefficients of the basis functions!

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(c) Use your results from part (b) to solve the following boundary value problem:

$$\begin{cases} \phi''(x) = f(x), \quad x \in [0,1]; \\ \phi(0) = \phi(1) = 0, \end{cases}$$

where f is piecewise defined as

$$f(x) = \begin{cases} x \quad \text{if} \quad x < 1/2; \\ 1 - x \quad \text{if} \quad x \geq 1/2. \end{cases}$$