HOMEWORK SET 2

MATH 415, SUMMER 2013

1. Reading

Finish reading sections 2.2 – 2.4 from the textbook. Read chapter 3 and start reading chapter 4.

2. Problems

Most of these problems are from Sadun's book.

(1) Section 2.3, problems 1–3, 6–8, 12.

(2) Section 2.4, problems 1, 2, 5, 8, 9, 12.

(3) Section 3.1, problems 5–7, 9–11, 13.

(4) Suppose $T : V \to W$ and $L : W \to Z$ are linear maps between vector spaces V, W and Z. Show that the composition map, $L \circ T : V \to Z$, defined by $L(T(\mathbf{v}))$ where $\mathbf{v} \in V$ is a linear mapping.

(5) Consider the mapping, $D : C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$ defined by $(Df)(x) = f'(x)$.
   (a) Show that D is a linear mapping. (You may use standard results from calculus).
   (b) Find all vectors $f \in C^\infty(\mathbb{R})$ such that $Df = 0$.
   (c) Because D is a linear *operator*, we can compose it with itself. Define $D^2 := D \circ D$. That is, $D^2 f = D(Df)$. According to exercise (4) this is a linear map. Find all vectors $f \in C^\infty(\mathbb{R})$ such that $D^2 f = 0$.
   (d) Now consider applying D a total of n-times. Find all vectors $f \in C^\infty(\mathbb{R})$ such that $D^n f = 0$.

(6) Section 3.2 problems, 1–4, 6 (c.f. exercise 4 - this set), 8, 11, 12.

(7) More coming soon!

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