Math 421	Homework 3	**Assigned**:	Thursday, January 30
Spring 2014	"Euclidean Space $\mathbb{R}^n$"	**Due**:	Friday, February 6

Directions:

- All problems should be collocated, stapled and contain a single cover sheet with your name and a list of problems.
- Clearly mark one problem that you would like to have graded. Additional problem(s) will be selected randomly.
- For each problem, submit only the final version of your solution. There should be enough details describing how you arrived at your solution, but do not submit "scratch-work" containing unnecessary computations.
- Homework is due at the *start of class* on the due date.

WARMUP PROBLEMS (Not to be turned in)

1. Texbook problems:

Section	Problem
8.1	8.1.1,
8.3	8.3.1
9.1	9.1.1, 9.1.2

HOMEWORK EXERCISES

1. Textbook exercises:

Section	Problem
8.1	8.1.2 (a, b),
8.3	8.3.2, 8.3.4, 8.3.8
9.1	9.1.3

2. Consider the following two functions from $\mathbb{R}^n$ to $\mathbb{R}$:

   (a) The l^1-norm, $\|\cdot\|_1 : \mathbb{R}^n \to \mathbb{R}$ defined by

$$\|\mathbf{x}\|_1 := \sum_{i=1}^{n} |x_i|.$$

   (b) The l^∞-norm, $\|\cdot\|_\infty : \mathbb{R}^n \to \mathbb{R}$ defined by

$$\|\mathbf{x}\|_\infty := \max_{1\le i\le n} \{|x_i|\}.$$

Show that each of them are *norms* on $\mathbb{R}^n$ by proving that they satisfy the three properties of a norm defined in class.

3. Show that $\|\cdot\|_\infty$ and $\|\cdot\|_1$ satisfy the following inequality on $\mathbb{R}^n$:

$$\|\mathbf{x}\|_\infty \leq \|\mathbf{x}\|_1 \leq n\|\mathbf{x}\|_\infty,$$

   and conclude that these two norms are equivalent.

4. Suppose $\sum_{k=1}^{\infty} a_k^2 < \infty$ and $\sum_{k=1}^{\infty} b_k^2 < \infty$. Show that the series $\sum_{k=1}^{\infty} a_k b_k$ converges absolutely.

5. Prove the following two results that are extensions of theorems from single-variable calculus:

   (a) Show that the limit of a convergence sequence in $\mathbb{R}^n$ is unique.

   (b) Every convergent sequence in $\mathbb{R}^n$ is bounded, but not conversely.