Math 421	Homework 2	**Assigned**:	Wednesday, January 22
Spring 2014	"The Riemann Integral, continued"	**Due**:	Wednesday, January 29

Directions:

- All problems should be collocated, stapled and contain a single cover sheet with your name and a list of problems.
- Clearly mark one problem which you would like to have graded. Additional problem(s) will be selected randomly.
- For each problem, submit only the final version of your solution. There should be enough details describing how you arrived at your solution, but do not submit "scratch-work" containing unnecessary computations.
- Homework is due at the *start of class* on the due date.

WARMUP PROBLEMS (Not to be turned in)

1. If $f : [a, b] \to \mathbb{R}$, give two equivalent definitions for what it means for f to be (Riemann) integrable.
2. Give a precise statement for what it means for f to be *continuous* on an interval $[a, b]$.
3. Give a precise statement for what it means for f to be *uniformly continuous* on an interval.
4. True/False. If $f(x) > 0$ for all $x \in I$, where I is some interval, then $\inf_{x\in I} f(x) > 0$.
5. Consider the partition $P = \left\{0, \frac{\pi}{2}, \pi\right\}$ of $[0, \pi]$. If $f(x) = \sin(x)$, compute the following:
   (a) $U(f, P)$ and $L(f, P)$.
   (b) $S(f, P, t_j)$, where the sampling is given by $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$.

HOMEWORK EXERCISES

1. Textbook exercises:

Section	Problem
5.2	5.2.5, 5.2.7, 5.2.11
5.3	5.3.5

2. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0. \end{cases}$$

Show that f is differentiable, but f' is not continuous on $\mathbb{R}$. *Hint:* You need to use the definition of the derivative at $x = 0$ to compute $f'(0)$.

3. For $n \in \mathbb{Z}_{\geq 0}$, define $g_n(x) = 2xne^{-nx^2}$ for $x \in [0, 1]$.

   (a) Show that for each $x \in [0, 1]$, we have $\lim_{n\to\infty} g_n(x) = 0$.

   (b) Using the Fundamental Theorem of Calculus to evaluate the integrals on the left, show that

$$\lim_{n\to\infty} \int_0^1 g_n(x)\, dx \neq \int_0^1 \left(\lim_{n\to\infty} g_n(x) \right) dx.$$

4. Suppose $f \in C^2([0, 1])$, the space of twice differentiable functions with continuous second-derivatives.

   If $f(0) = f'(1) = 0$ and $\int_0^1 f(x) f''(x)\, dx = 0$, show that $f \equiv 0$.