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12.) Prove that the following is partially correct wrt the initial assertion "a and d are positive integers" and the final assertion "q and r are integers such that $a = dq + r$ and $0 \le r < d$".

Proof Consider the loop invariant

$p =$ "$a = dq + r$, and $r \ge 0$".

When the program executes, we set $r = a$ and $q = 0$, so p is true because the initial assertion is true, and

$a = d \cdot 0 + r$.

Inside the loop, the first assignment replaces r with $r - d$. because $r \ge d$, we know $r - d \ge 0$, so r remains positive. q gets replaced with $q+1$, but we know

$a = dq + r = d(q+1) + (r-d)$

So the loop invariant remains true.

~~After the loop is~~

Because r is decremented by a positive amount, the loop will terminate.

After termination, we have

$$\neg \text{Condition} \wedge P \equiv \neg(r \geq d) \wedge P$$

$$\equiv (r<d) \wedge P$$

$$\equiv (r<d) \wedge \text{"}a=dq+r \wedge r \geq 0\text{"}$$

$$\equiv (r<d) \wedge \text{"}a=dq+r\text{"} \wedge (r \geq 0)$$

$$= (0 \leq r<d) \wedge \text{"}a=dq+r\text{"}$$