SECTION 2.3

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If $f : X \to Y$ is a function, we say that X is the *domain*, and Y is the *co-domain*. The *range* of f is defined as the set

$$f(X) = \{y \in Y | \exists x \in X, f(x) = y\}.$$

In general, if $A \subseteq X$ is a subset of X, we define the *image* of A as the set

$$f(A) = \{y \in Y | \exists x \in A, f(x) = y\}.$$

The *image* of a set A is oftentimes abreviated with $f(A) = \{f(x) | x \in A\}$.

Here are a couple of questions to think about.

a.) Does the range equal the codomain? (The answer to this should require next to no thought) If not, provide an example of a function, a domain, and a codomain where the range doesn't equal the codomain.

b.) If the answer to (a) is false, what is the word for a function where the codoman and range equal each other?

2–3, #4 Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.

a.) the function that assignes to each nonnegative integer its last digit.

Domain $= \mathbb{N}$ becuase it takes nonnegative integers as input. The Range is the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, assuming we are counting numbers in base 10.

b.) The function that assigns the next largest integer to a positive integer.

Domain $= \mathbb{N} - \{0\} = \{1, 2, 3, 4, \ldots\}$. The range is $\mathbb{N} - \{0, 1\} = \{2, 3, 4, \ldots\}$.

c.) The function that assigns to a bit string the number of one bits in the string.

Here we need to decide what we mean by a bit string. Usually, this means strings of finite length. For example, we would like to include 0101 as well as 11010010. They have different lengths, but nontheless we want the function to be able to do something with both of these. One such way of representing strings of length d is to define the set $S_d = \{0, 1\}^d$ as the cartesian product of $\{0, 1\}$ with itself d times. In order to look at all possible sets of this form, we consider the union of all of these. In other words, the domain is the set

$$X = \bigcup_{d=1}^{\infty} S_d = \bigcup_{d=1}^{\infty} \{0, 1\}^d.$$

The range is all possible output values of this function. In this case, the range is simply $\mathbb{N}$. If we consider the codomain as $\mathbb{N}$, then this function is onto. If on the other hand we had considered the codomain as $\mathbb{R}$ or $\mathbb{Z}$, then this function would not be onto.

Here's a question to think about. Is this function one-to-one? If so, prove it. If not, show that it isn't one-to-one. In order to do this, find an $x \neq y \in X$, with $f(x) = f(y)$.

d.) The function that assigns to a bit string the number of bits in the string.

The domain is the same as in part (c). The range would be $\mathbb{N} - \{0\}$, since with our choice of the domain, the smallest string has length 1.