Math 299
Fall 2014

Homework 7

Due: Monday, October 13.

Directions:

- Print out this piece of paper and use it as a cover sheet. Write your name in the upper right hand corner.
- Your homework should be stapled and each problem should occur in order.
- Do not hand in scratch work.

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1. Problems

1. Finish reading Chapter 5. Start reading Chapter 6.
2. Let $R_0 = \{x \in \mathbb{Z} : x \equiv 0 \mod 3\}$, $R_1 = \{x \in \mathbb{Z} : x \equiv 1 \mod 3\}$, and $R_2 = \{x \in \mathbb{Z} : x \equiv 2 \mod 3\}$. Determine if the following statements are true or false, and justify your answer with a proof.
   (a) $\forall n \in R_0, n^2 \in R_0$.
   (b) $\forall n \in R_1, n^2 \in R_2$.
   (c) $\forall n \in R_2, n^2 \in R_2$.
   (d) $\forall n \in \mathbb{Z}, n^2 \notin R_2$.
   (e) $\forall n \in R_1 \cup R_2, \exists m \in \mathbb{Z}, nm \in R_1$.
3. Let $\{B_x\}_{x \in S}$ be an indexed family of sets, suppose $T \subseteq S$.
   (a) Prove that $\cup_{x \in T} B_x \subseteq \cup_{x \in S} B_x$.
   (b) Prove, by providing a counterexample, that it is not necessarily the case that it is not always the case that $\cup_{x \in T} B_x = \cup_{x \in S} B_x$.
   (c) Suppose that $T \neq S$. Does it follow that $\cup_{x \in T} B_x \neq \cup_{x \in S} B_x$? Justify your answer.
4. Textbook exercises: 5.4, 5.6, 5.16, 5.28 (*Hint:* if m and n are integers and $m > n$, then $m \geq n + 1$.)
