Math 299
Fall 2014

Homework 15

Due: Wednesday, Nov. 26

Directions:

- Print out this piece of paper and use it as a cover sheet. Write your name in the upper right hand corner.
- Your homework should be stapled and each problem should occur in order.
- Do not hand in scratch work.
- Homework is due at the start of class.

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1. Problems

1. Textbook exercises: 12.49, 12.55.
2. If a_n is a sequence with $\lim_{n\to\infty} a_n = 0$, prove that $\lim_{n\to\infty} a_n^2 = 0$.
3. Suppose $\{a_n\}$ is a sequence of positive real numbers that converges to a. Prove that $a \geq 0$. *Hint:* try a proof by contradiction.
4. Suppose $\{a_n\}$ is a sequence of positive real numbers, and that $\lim_{n\to\infty} a_n = a > 0$. Prove that $\lim_{n\to\infty} \sqrt{a_n} = \sqrt{a}$. *Hint:* "irrationalize the denominator."
5. Define the sequence $a_1 = 1$, and $a_{n+1} = 5a_n^4$ for all $n \geq 1$.
   (a) If $a = \lim_{n\to\infty} a_n$, exists, prove that $a = 0$ or $a = 1/5$.
   (b) Does $a = \lim_{n\to\infty} a_n$ exist?
   (c) Do parts (a) and (b) present a contradiction?
6. Find an example of a convergent sequence $\{s_n\}$ of irrational numbers that has a rational number as a limit.
