Math 451	Homework 4	**Assigned**:	Monday, October 28
Fall 2013	"Interpolation: Part I"	**Due**:	Wednesday, November 6

Directions:

- Include a cover page.
- For each problem, submit only the final version of your solution. Each problem should be collocated and stapled.
- Always label every plot (title, x-label, y-label, and legend).
- Use the subplot command from MATLAB (or matplotlib) when comparing 2 or more plots to make comparison easier and to save paper.

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1. Consider the function
$$f(x) = \sin\left(\frac{\pi}{2}x\right) + \frac{x}{2}.$$
We would like to construct a polynomial $P(x)$ that interpolates $f(x)$ at the points $x_0 = 0$, $x_1 = 2$, and $x_2 = 3$.
   (a) Construct the Lagrange interpolating polynomial, $P_2(x)$.
   (b) Compute the error term $R(x)$.
   (c) Plot the difference between the polynomial from part (a) and the function $f(x)$. Use x in the range $[0, 3]$.
   (d) Establish a theoretical upper bound for the interpolating polynomial.
   (e) Use the interpolating polynomial to estimate $f(0.5)$ and $f(1.0)$. Compare with the theoretical upper bound.

2. Suppose you are given a divided differences table with missing data:

$x_0 = 0$	$f[x_0] = 1$			
		$f[x_0, x_1] = 2$		
$x_1 = 1$	$f[x_1] = 3$		$f[x_0, x_1, x_2] =?$	
		$f[x_1, x_2] =?$		$f[x_0, x_1, x_2, x_3] =?$
$x_2 = 2$	$f[x_2] = 3$		$f[x_1, x_2, x_3] = 0$	
		$f[x_2, x_3] = 0$		
$x_3 = 3$	$f[x_0] =?$			

   (a) Determine the missing values in the divided difference table.
   (b) From the divided differences table, write down, by hand, the interpolating polynomial that interpolates the above node and function values.

3. Construct the divided difference table of the following data set, and then write out the Newton form of the interpolating polynomial.

x	0	1	3
y	1	-1	5

4. Construct the divided difference table of the following data set, and then write out the Newton form of the interpolating polynomial.

x	-7	-5	-4	-1
y	10	5	2	10

5. **SOURCE CODE:**

Write the following two MATLAB routines:

- **F = DivDiff(xnd, f)** - a function to compute the Newton form of the interpolating polynomial for function values **f** through the nodes provided by **xnd**. The distinct nodes are given by

$$\mathbf{xnd}(1) = x_0, \quad \mathbf{xnd}(2) = x_1, \quad \ldots, \quad \mathbf{xnd}(n+1) = x_n$$

are not necessarily in order. The returned vector **F** should save the coefficients of the Newton form of the interpolating polynomial in terms of divided differences:

$$\mathbf{F}(1) = f[x_0], \quad \mathbf{F}(2) = f[x_0, x_1], \quad \ldots, \quad \mathbf{F}(n+1) = f[x_0, x_1, x_2, \ldots, x_n].$$

- **P = EvalDivDiff(x, xnd, F)** - A function that evaluates the Newton form of the divided differences at a list of points. Here, **xnd** is the vector containing each point used to construct **F**, and **x** is a list of points to evaluate the polynomial.

6. **EXPLORATION PROBLEM:**

Consider the function

$$f(x) = \frac{1}{1+25x^2}.$$

(a) Let x_i be 11 equally spaced points on $[-1, 1]$:

$$x_i = -1 + i/5, \quad \text{for} \quad i = 0, 1, 2, \ldots, 10.$$

(b) Use your code from the previous problem to construct the divided differences with these x_i.

(c) Sample the polynomial at 500 uniformly chosen points, $\mathbf{z} = \mathbf{linspace}(-1, 1, 500)$, by calling $\mathbf{P} = \mathbf{EvalDivDiff}(\mathbf{z}, \mathbf{xnd}, \mathbf{F})$

(d) Make a single MATLAB plot that contains: $(z, P), (z, f(z))$, and the data points $(x_i, f(x_i))$ for $i = 0, 1, 2, \dots 10$. Make the $P(z)$ blue solid line, $f(z)$ a red dashed line, and use x's, dots or stars for the data points.

(e) Now, let xi be 11 unequally spaced points on $[-1, 1]$:

$$x_i = \cos\left(\frac{\pi(i + 0.5)}{11}\right), \quad \text{for} \quad i = 0, 1, 2, \dots, 10.$$

Repeat steps (a)–(c) for these unequally spaced data points.

(f) Explain what you observe.

7. **APPLICATION PROBLEM:**

Consider the following set of data relating the thermal conductivity of air $(mW/m \cdot K)$ as a function of temperature (K):

Temperature (K)	100	200	300	400	500	600
Thermal Conductivity $(mW/m \cdot K)$	9.4	18.4	26.2	33.3	39.7	45.7

(a) Apply your divided differences code to obtain the coefficients of the Newton form of the interpolating polynomial.

(b) Evaluate the interpolating polynomial to estimate the thermal conductivity of air when $T = 240K$ and when $T = 485K$.