Math 451	Homework 3	**Assigned**:	Monday, September 30
Fall 2013	"Solving linear systems: Direct Solvers"	**Due**:	Friday, October 11

Directions:

- Include a cover page.
- For each problem, submit only the final version of your solution. Each problem should be collocated and stapled.
- Always label every plot (title, x-label, y-label, and legend).
- Use the subplot command from MATLAB (or matplotlib) when comparing 2 or more plots to make comparison easier and to save paper.

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1. Without using pivots, write out, by hand, each step in the LU decomposition for

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & 2 & 3 \\ 1 & -1 & 1 & 2 \\ -1 & 1 & -1 & 5 \end{pmatrix}.$$

2. **SOURCE CODE:**

   Write the following functions in MATLAB:

   (a) Warmup:
   - **x = BackwardSubs(U, y)** - backward substitution
   - **y = ForwardSubs(L, b)** - forward substitution

   (b) A first pass:
   - **x = GaussElim(A, b)** – Naive Gaussian elimination (no pivoting)
   - **[L, U] = LuDecomp(A)** - LU decomposition (no pivoting)

   (c) The third time's the charm:
   - **x = GaussElimPP(A, b)** – Gauss elimination with partial pivoting
   - **[L, U, P] = LuDecompPP(A)** - LU decomposition with partial pivoting

   Each of these functions should be saved in a single file. For example, place the title

   function **x = BackwardSubs(U, y)**

   as the first line in a file called **BackwardSubs.m**.

   As always, your scripts will be graded for correctness as well as completeness of documentation and cleanliness of code. **These codes are to be submitted as part of your assignment, by email**. Use the subject line: "MTH 451 HW3 Scripts" in your email.

3. Test the two naive solvers written in 2(b) on the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = (10, 5, 3, 4)^T$, and A is the matrix given in problem 1. Use your code from 2(a) to perform the forward and backward substitutions.

4. Test the two solvers written in 2(c) on the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = (-53, 18, -7, 0, -103)^T$ and
$$A = \begin{pmatrix} 13 & 39 & 2 & 57 & 28 \\ -4 & -12 & 0 & -19 & -9 \\ 3 & 0 & -9 & 2 & 1 \\ 6 & 17 & 9 & 5 & 7 \\ 19 & 42 & -17 & 107 & 44 \end{pmatrix}$$
Do your solvers from 2(b) work on this problem? Why or why not?

5. $\boxed{\textbf{APPLICATION:}}$

The *inverse power method* is an iterative numerical method to find the smallest eigenvalue of a matrix. It is given by the following updates:
$$\begin{aligned} \mathbf{x}^{(m+1)} &= (A - \lambda_0 I)^{-1}\mathbf{x}^{(m)}, \\ \lambda_m &= x^{(m)}_{p_{m-1}}, \\ \mathbf{x}^{(m)} &= \mathbf{x}^{(m)}/x^{(m)}_{p_m}. \end{aligned}$$
The quantity $\lambda_0 + (1/\lambda_m)$ converges to the eigenvalue of A that is closest to λ_0, and $\mathbf{x}^{(m)}$ converges to the corresponding eigenvector. The integer p_m is chosen so that $|x^{(m)}_{p_m}| = \|\mathbf{x}^{(m)}\|_\infty$. For the remainder of this exercise, let
$$A = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{pmatrix}.$$

(a) Select $\lambda_0 = 6$ and $\mathbf{x}^{(0)} = (1, 1, 1)^T$. How does $\lambda_0 + 1/\lambda_5$ compare with the exact eigenvalue, $\lambda \approx 5.124885419764584$?

(b) Select $\lambda_0 = -1$ and $\mathbf{x}^{(0)} = (1, 1, 1)^T$. How does $\lambda_0 + 1/\lambda_5$ compare with the exact eigenvalue, $\lambda \approx 0.238442818168109$?

In place of inverting the system at each step, as a pre-processing step, construct a single LU decomposition on the matrix $T = (A - \lambda_0 I)$. Recall that the decomposition will give you $PT = LU$, so you'll have to reorder the right hand side vector at each step:
$$T\mathbf{x} = \mathbf{b} \implies PT\mathbf{x} = LU\mathbf{x} = P\mathbf{b}.$$
As part of your assignment, include a single, self-contained script, **run_inverse_power_method.m** that calls your functions written in problem 2.