309 Worksheet 3.2

(1) *Restate the definition of linear independence/dependence using quantifiers 'for all' and 'exists'.*

(2) *Write each statement below as a statement using 'for all' and 'exists':*

(a) For every positive real number ϵ, there is a natural number n with $\frac{1}{n} < \epsilon$.

(b) Every even integer greater than 2 is the product of an even integer and a prime number.

(c) For every positive real number ϵ, there is a positive number δ such that $x^2 < \epsilon$ whenever $|x| < \delta$.

(d) There exists an integer m with the property that for every integer x, there is an integer y with $xy = m$.

(e) There is always some prime number strictly between any given integer $n > 1$ and its square.

(3) *Determine whether each statement below is true or false. Give the negation of each statement:*

(a) For all $x \in \mathbb{R}$ there is an $a \in \mathbb{R}$ with $|x| < a$

(b) There is an $a \in \mathbb{R}$ such that for all $x \in \mathbb{N}, a < x$

(c) For all $x \in \mathbb{R}$ there is a $y \in \mathbb{R}$ such that $xy = 1$

(d) There is a real number $b \in \mathbb{R}$ such that for all $a \in \mathbb{N}, |a - b| \leq 100$

(e) For all $a \in \mathbb{R}, \sqrt{a^2} = a$

(f) For all $a \in [0, \infty)$ there is an $x \in \mathbb{R}$ such that $x^2 = a$ and $-x^2 = a$

(g) There is an integer $x \in \mathbb{Z}$ such that for all $y \in \mathbb{Z}, \frac{y}{x} \in \mathbb{Z}$

(h) For all $a \in \mathbb{N}$ there are integers $b, c \in \mathbb{N}$ such that $ab = c^3$