# Test 1 Practice Test

Math 309, Section 6

1. Consider the following system of linear equations:

$$\begin{aligned} -3y + z &= 1 \\ x + y - 2z &= 2 \\ x - 2y - z &= 3 \end{aligned}$$

Write the coefficient matrix associated to the linear system. Use Gaussian elimination (and write what elementary row operations you use) to put the matrix into reduced echelon form. Write the solution set to the system of linear equations. Write the solution set as a line or plane in a vector space (in what vector space does the solution set live?).

2. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a finite subset of the vector space $V$. Write the definition of span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. What does it mean for $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ to span $V$? Give the definition of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ being linearly independent. By definition, what does it mean for $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ to be a basis?

3. Is $\{\mathbf{0}\}$ linearly independent? Justify your answer.

4. (a) Suppose that $\{\mathbf{v}, \mathbf{w}\}$ are linearly independent in $V$, and $\mathbf{x} \in V$. Then, is $\{\mathbf{v}, \mathbf{w}, \mathbf{x}\}$ linearly independent? Yes, no, maybe?

   (b) Suppose $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ spans $V$. Show that $\{\mathbf{v}_1, \dots, \mathbf{v}_n, \mathbf{v}_{n+1}\}$ also spans $V$.

5. Is the set of polynomials $\{x^2, 4x^2 - 2, 1\}$ linearly independent in $\mathbb{P}_2$? If not, find a subset of $\{x^2, 4x^2 - 2, 1\}$ which is linearly independent.

6. Which of the following are subspaces? What properties of a subspace do they satisfy or not satisfy? (Here, $p(a)$ means the polynomial evaluated at $x = a$; e.g. if $p = x^2 - 2$, then $p(1) = 1^2 - 2 = -1$.)

$$\{f \in \mathbb{D}^{(2)}(\mathbb{R}) \mid x^4 f'' - f^2 = 0\} \subset \mathbb{D}^{(2)}(\mathbb{R})$$
$$\{f \in \mathbb{D}^{(2)}(\mathbb{R}) \mid x^4 f'' - e^{-x^2} f = 0\} \subset \mathbb{D}^{(2)}(\mathbb{R})$$
$$\{p \in \mathbb{P}_n \mid p(1) = 0\} \subset \mathbb{P}_n$$

7. Suppose $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for $V$. Show that $\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_2, \mathbf{v}_3\}$ is also a basis for $V$.

8. Consider the subspaces $V_1$ and $V_2$ of $\mathbb{R}^3$ defined by

$$\begin{aligned} V_1 &= \{(x, y, z) \mid -3y + z = 0\} \\ V_2 &= \{(x, y, z) \mid x + y - 2z = 0\} \end{aligned}$$

Find a basis for $V_1 \cap V_2$.

9. Listed here are the 8 axioms of a vector space:

| | |
|---|---|
| 1. $\mathbf{v}+\mathbf{w}=\mathbf{w}+\mathbf{v}$ | (addition commutative) |
| 2. $(\mathbf{v}+\mathbf{w})+\mathbf{x}=\mathbf{v}+(\mathbf{w}+\mathbf{x})$ | (addition associative) |
| 3. $\exists\,\mathbf{0}\in V$ such that $\mathbf{v}+\mathbf{0}=\mathbf{v}$ for all $\mathbf{v}$ | (additive identity) |
| 4. $\forall\,\mathbf{v}\in V$, $\exists\,(-\mathbf{v})\in V$ such that $\mathbf{v}+(-\mathbf{v})=\mathbf{0}$ | (additive inverse) |
| 5. $r(\mathbf{v}+\mathbf{w})=r\mathbf{v}+r\mathbf{w}$ | (distributive) |
| 6. $(r+s)\mathbf{v}=r\mathbf{v}+s\mathbf{v}$ | (distributive) |
| 7. $r(s\mathbf{v})=(rs)\mathbf{v}$ | (scalar associative) |
| 8. $1\mathbf{v}=\mathbf{v}$ | (scalar identity) |

Using only vector space axioms, show the following properties of vector spaces (justify all your steps):

(a) $\mathbf{v}+(\mathbf{0}+-\mathbf{v})=\mathbf{0}$

(b) If $\mathbf{v}+\mathbf{w}=\mathbf{0}$, then $\mathbf{w}=-\mathbf{v}$.

10. Let $S=\{ax^2+bx+c\in\mathbb{P}_2 \mid a+b-2c=0\}\subset\mathbb{P}_2$.

(a) Show $S$ is a subspace of $\mathbb{P}_2$.

(b) Find a basis for $S$.

(c) What is the dimension of $S$?

11. (a) Is $\{(1,2,3),(0,1,7),(-1,4,-8),(3,0,4)\}$ a set of linearly independent vectors in $\mathbb{R}^3$?

(b) Does $\{(1,1,0),(1,0,1),(0,1,1)\}$ span $\mathbb{R}^3$?