HOMEWORK DUE WEDNESDAY 12/1

MATH 309, SECTION 3

(1) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given in standard coordinates by the matrix

$$\begin{bmatrix} -1 & 6 \\ \frac{3}{2} & -1 \end{bmatrix}$$

Let $B = \{(1,0),(0,1)\}$ and $B' = \{(-2,1),(2,1)\}$. Find the change of basis matrices $P_{BB'}$ and $P_{B'B}$ and use these to compute the matrix of T relative to B'.
(i.e. the above matrix is T_{BB}, and use $P_{BB'}$ and $P_{B'B}$ to find $T_{B'B'}$).

(2) Let $T : \mathbb{P}_2 \to \mathbb{R}^2$ be given by

$$T(p) = \begin{bmatrix} p(0) \\ p(2) \end{bmatrix}$$

(e.g. if $p = a + bx$, then $p(4) = a + b(4) = a + 4b$.)
   (a) Find the matrix of T relative to the standard bases $B = \{1, x, x^2\}$ of $\mathbb{P}_2$, and $C = \{\mathbf{e}_1, \mathbf{e}_2\}$ of $\mathbb{R}^2$.
   (b) Find the matrix of T relative to the basis $A = \{1, 1+x, 1+x+x^2\}$ of $\mathbb{P}_2$ and $D = \{(1,1),(1,-1)\}$ of $\mathbb{R}^2$.

(3) Suppose that $T : \mathbb{P}_2 \to \mathbb{P}_2$ satisfies

$$T(1+x) = 3(x+x^2), \quad T(x+x^2) = -(x^2+1), \quad T(x^2+1) = 2(1+x).$$

Find the matrix of T relative to the basis $\{1+x, x+x^2, x^2+1\}$, and use change of basis matrices to find the matrix of T relative to the standard basis of $\mathbb{P}_2$.

(4) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ satisfies

$$T(3,1) = 5(3,1), \quad T(0,2) = -1(0,2).$$

Find the matrix of T relative to the standard basis of $\mathbb{R}^2$.