HW ASSIGNED 10/27

MATH 309, SECTION 3

(1) Show that if $T : V \to W$ is a linear map, then the image of T

$$\mathrm{Im}(T) = \{\mathbf{w} \in W \bigm| \mathbf{w} = T(\mathbf{v}) \text{ for some } \mathbf{v} \in V\}$$

is a subspace of W.

(2) Given any finite set of vectors $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ define the map $T : \mathbb{R}^n \to V$ by

$$T(r_1, \dots, r_n) = r_1\mathbf{v}_1 + r_2\mathbf{v}_2 + \dots r_n\mathbf{v}_n.$$

Verify that this map is linear.

(3) If $S, T : V \to W$ are linear maps, show that $(r_1 S + r_2 T)$ is linear for any $r_1, r_2 \in \mathbb{R}$.

(4) Write the following subspaces as either the kernel or image of a linear map:

(a) $\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{M}(2,2) \bigm| 2a + b = 0 \right\}$

(b) $\{a(x^2 + 2) + b(x^2 - 1) + c(x + 1) \in \mathbb{P}_2 \bigm| a, b, c \in \mathbb{R}\}$

(c) $\{f \in \mathbb{C}([0,1]) \bigm| \int_0^1 f(x)dx = 0\}$

(d) Solutions (x, y, z) to $\begin{cases} x - y + 7z = 0 \\ 2x - y + 11z = 0 \\ -x - y + 3z = 0 \end{cases}$

(e) The set of all functions $g \in \mathbb{C}[0,1]$ for which a solution exists to the differential equation

$$y'' - ky = g,$$

where $y = y(x) \in \mathbb{D}^{(2)}([0,1])$ and $k \in \mathbb{R}$.