HW ASSIGNED 10/18

MATH 309, SECTION 3

(1) (4.4: 4,5) Verify that $\{(3,6,-2),(-2,3,6),(6,-2,3)\}$ is an orthogonal subset of $\mathbb{R}^3$ (with dot product as inner product). Show that $\{(\frac{3}{7},\frac{6}{7},-\frac{2}{7}),(-\frac{2}{7},\frac{3}{7},\frac{6}{7}),(\frac{6}{7},-\frac{2}{7},\frac{3}{7})\}$ is an orthonormal subset of $\mathbb{R}^3$.

(2) If $\{\mathbf{v}_1,\dots,\mathbf{v}_n\}$ is a set of non-zero orthogonal vectors in V, how can you produce an a set of n orthonormal vectors (i.e. orthogonal with norm 1)?

(3) Show that $\{\sin x,\cos x,1\}$ is orthogonal in $\mathbb{C}[-\pi,\pi]$.
Reminder: You need to show that ($\sin x$ and $\cos x$ are orthogonal), ($\sin x$ and 1 are orthogonal), and ($\cos x$ and 1 are orthogonal).

(4) If $\{\mathbf{v}_1,\dots,\mathbf{v}_n\}$ is a set of non-zero orthogonal vectors in an inner product space V, prove that $\{\mathbf{v}_1,\dots,\mathbf{v}_n\}$ are linearly independent.
Hint: Assume solution $\sum_i r_i\mathbf{v}_i=\mathbf{0}$, which then implies
$$\langle\sum_i r_i\mathbf{v}_i,\sum_j r_j\mathbf{v}_j\rangle=\langle\mathbf{0},\sum_j r_j\mathbf{v}_j\rangle.$$
Use above equation to show $r_1=r_2=\cdots=r_n=0$.

(5) *Remark: Given any basis of V, one can define an inner product on V by declaring the basis to be orthonormal.*
Let $\langle,\rangle$ be an inner product on $\mathbb{R}^2$ so that $\{(1,1),(0,1)\}$ is an orthonormal basis. (Note that this inner product is different than the dot product, but it still satisfies the axioms of being an inner product.) Compute the inner product of two arbitrary vectors $\langle(x_1,y_1),(x_2,y_2)\rangle$.
Hint: First write a vector (x,y) as a linear combination of $(1,1)$ and $(0,1)$. Use fact that
$$\langle(1,1),(1,1)\rangle=\langle(0,1),(0,1)\rangle=1,\quad\langle(1,1),(0,1)\rangle=0.$$