Test 4 Review

1. Find the length of the curve $\mathbf{r}(t) = (\cos^3 t)\mathbf{i} + (\sin^3 t)\mathbf{k}$, $0 \le t \le \pi/2$.

   Solution: $3/2$

2. A wire is given by the straight-line segment connecting $(0,1,0)$ to $(1,0,0)$ and has density $\delta = x + y$. Find the mass of the wire.

   Solution: $\sqrt{2}$

3. Calculate the work done by the force $\mathbf{F} = \langle 3x^2 - 3x, 3z, 1\rangle$ from $(0,0,0)$ to $(1,1,1)$ over the path $\mathbf{r}(t) = \langle t, t^2, t^4\rangle$, $0 \le t \le 1$.

   Solution: $3/2$

4. Let $\mathbf{F} = \sin y \cos x\mathbf{i} + \cos y \sin x\mathbf{j} + \mathbf{k}$. Show that $\mathbf{F}$ is conservative and find a potential function. Calculate the integral
$$\int_C \sin y \cos x dx + \cos y \sin x dy + dz$$
   where C is the straight-line from $(1,0,0)$ to $(0,1,1)$.

   Solution: $\mathbf{F} = \nabla(\sin x \sin y + z + C)$; 1

5. Use Green's Theorem to evaluate
$$\oint (6y + x)dx + (y + 2x)dy$$
   where C is the (counter-clockwise) circle $(x-2)^2 + (y-3)^2 = 4$.

   Solution: -16π

6. Find the area of the surface $x^2 - 2y - 2z = 0$ that lies above the triangle bounded by the lines $x = 2, y = 0, y = 3x$ in the xy-plane.

   Solution: $6\sqrt{6} - 2\sqrt{2}$

7. Integrate $g(x,y,z) = x + y + z$ over the portion of the plane $2x + 2y + z = 2$ that lies in the first octant.

Solution: 2

8. Find the flux of $\mathbf{F} = \langle x, y, z \rangle$ across the portion of the sphere $x^2 + y^2 + z^2 = 9$ in the first octant in the direction away from the origin.

Solution: $\frac{27\pi}{2}$

9. Use Stokes' Theorem to calculate the work done by $\mathbf{F} = x^2\mathbf{i} + 2x\mathbf{j} + z^2\mathbf{j}$ around the ellipse $4x^2 + y^2 = 4$ in the xy-lane, counterclockwise when viewed from above.

Solution: 4π

10. Use Stokes' Theorem to calculate the work done by $\mathbf{F} = \langle yz, 2xz, 0 \rangle$ around the closed curve C given by the intersection of the cylinder $x^2 + y^2 = 1$ with the plane $z = y + 1$. The curve C is counter-clockwise when viewed from above.

Solution: π

11. Use Stokes' Theorem to evaluate $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, d\sigma$ where S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$ oriented upward, and $\mathbf{F} = x^2 e^{yz}\mathbf{i} + y^2 e^{xz}\mathbf{j} + z^2 e^{xy}\mathbf{k}$.

Solution: 0

12. Use the Divergence Theorem to find the outward flux of $\mathbf{F} = \langle y, xy, -z \rangle$ across the boundary of the region inside the solid cylinder $x^2 + y^2 \leq 4$, between the plane $z = 0$ and the paraboloid $z = x^2 + y^2$.

Solution: -8π

13. Let $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and suppose that the surface S and the region D satisfy the hypotheses of the Divergence Theorem. Show that the volume of D is given by the formula

$$\text{Volume of } D = \frac{1}{3} \iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma.$$