Test 3 Review

1. Evaluate the following integrals (you may need to switch order of integration):
   (a) $\int_0^\pi \int_0^x x \sin y \, dy \, dx$
   (b) $\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} \, dy \, dx$
   (c) $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (x^2+y^2) \, dy \, dx$
   (d) $\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2} \, dy \, dx$
   (e) $\int_0^1 \int_0^{2-x} \int_0^{2-x-y} dz \, dy \, dx$
   (f) $\int_{-1}^1 \int_0^{\sqrt{1-y^2}} \int_0^x (x^2+y^2) \, dz \, dx \, dy$

2. Set up integrals to integrate over the following regions:
   (a) the solid whose base is the region in the xy-plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $z = x + 4$.
   (b) the (2-dimensional) region bounded by $y = e^x$ and the lines $y = 0, x = 0, x = \ln 2$.
   (c) the (2-dimensional) region bounded by the parabolas $x = y^2 - 1$ and $x = 2y^2 - 2$.
   (d) the region cut from the first quadrant by the cardiod $r = 1 + \sin\theta$.
   (e) the tetrahedron cut from the first octant by the plane $6x + 3y + 2z = 6$.
   (f) the region bounded by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.
   (g) the region bounded by the cylinder $z = y^2$, and the planes $z = 0, x = 0, x = 1, y = -1, y = 1$.
   (h) (using cyclindrical coordinates) region bounded by $z = \sqrt{x^2+y^2}$, $z = 0$ and cylinder $(x-2)^2+y^2 = 4$.
   (i) (using spherical coordinates) cone with $z \geq 0$ formed by $z^2 = 2(x^2+y^2)$ and $z = 5$.
   (j) (using spherical coordinates) the portion of the sphere of radius 2, centered at the origin, satisfying $x \geq 0$.

3. Suppose the temperature at a point is given by the function $f = y$. Find the average temperature on the part of a ball of radius 2 in the first octant $(x, y, z > 0)$.

4. Suppose the density at a point is given by the function $\delta = x^2 + y^2$. Find the total mass and the center of mass of a disc of radius 1 in the xy-plane.

5. Find $\iint_R xy \, dA$ where R is the region bounded by the lines $y = x$, $y = 2x$, and $x + y = 2$.