Test 2 Review

1. Suppose a particle moving in three-dimensional space has position given by

$$\mathbf{r}(t) = \langle 2\cos t, 3\sin t, 4t\rangle, \quad t \geq 0.$$

   Find the velocity and speed at time $t = \frac{\pi}{2}$.

2. A particle is moving with velocity given by $\mathbf{v}(t) = \langle \frac{3}{2}\sqrt{1+t}, e^{-t}, \frac{1}{1+t}\rangle$ and initial position $\mathbf{r}(0) = \langle 3, -4, 1\rangle$. Find the position function $\mathbf{r}(t)$.

3. Find the length of the curve

$$\mathbf{g}(t) = (t\cos t)\mathbf{i} + (t\sin t)\mathbf{j} + (\frac{2\sqrt{2}}{3}t^{3/2})\mathbf{k}, \quad 0 \leq t \leq \pi.$$

4. Let $f(x,y) = \sqrt{16 - x^2 - y^2}$. Find the domain of $f(x)$ and sketch the level curves for $f = 0, 1, 2, 3, 4$.

5. Compute the following limits:
   (a) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{x+y}{x-y}$
   (b) $\displaystyle\lim_{(x,y)\to(2,2)} \frac{x+y}{x-y}$
   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x+y}{x-y}$
   (d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4+y^2}{x^4-y^2}$
   (e) $\displaystyle\lim_{\substack{(x,y)\to(4,3)\\ x\neq y+1}} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1}$

6. Calculate $f_x, f_y, f_{xx}, f_{xy}, f_{yx}$, and f_{yy}.
   (a) $f(x,y) = e^{xy}\ln y$
   (b) $f(x,y) = \sin^2(3x-5y)$

7. Find $\frac{dw}{dt}$ at $t = 3$ where $w = \frac{x}{z} + \frac{y}{z}$, $x = \cos^2 t$, $y = \sin^2 t$, $z = 1/t$.

8. If $f(u,v,w)$ is a differentiable function, and $u = x - y$, $v = y - z$, $w = z - x$, show that

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0.$$

9. Let $f(x,y)$ be a differentiable function. Using the polar coordinate transformations $x = r\cos\theta$, $y = r\sin\theta$, express $\dfrac{\partial f}{\partial \theta}$ purely in terms of the rectangular coordinates (x,y).

10. Calculate the directional derivative of $g(x,y,z) = x^2 + 2y^2 - 3z^2$ at the point $P_0(1,1,1)$ in the direction $\langle 1, 1, 1\rangle$.

11. Consider the curve in the plane given by the equation $x^2 - xy + y^2 = 7$. Find the equation of the tangent line at $(-1, 2)$.

12. Find the equation of the tangent plane to the surface $x^2 - xy - y^2 = z$ at the point $(1, 1, -1)$.

13. Using linear approximation, estimate $e^{.1}\sin(-.2)$.

14. Find all local extrema and saddle points for the following functions

    (a) $f(x, y) = x^2 + xy + 3x + 2y + 5$

    (b) $f(x, y) = 8x^3 + y^3 + 6xy$

15. Find the absolute maximum and minimum values of $f(x, y) = 1 + xy - x - y$ on the closed region bounded by the curves $y = x^2$ and $y = 4$.