Name: ANSWER KEY Section: ____________

Math 234: Sections 1-3

Test 1

February 2, 2010

The exam is all partial credit. Please write neatly and clearly, showing all of your work. No calculators, cell phones, books, or notes may be used. The test contains 100 possible points. Good luck!

1	15
2, 3	15
4	10
5, 6	20
7	13
8	20
9	7
Total	100

1. (15 points) Compute the following:

a. $(\mathrm{i}+\mathrm{j}-2\mathrm{k})+5(-\mathrm{i}+2\mathrm{k})$

$$= (\hat{\imath} + \hat{\jmath} - 2\hat{k}) + (-5\hat{\imath} + 10\hat{k})$$

$$= \boxed{-4\hat{\imath} + \hat{\jmath} + 8\hat{k}}$$

$$= \langle -4, 1, 8 \rangle$$

b. $\langle 3,-1,0\rangle \bullet \langle 2,3,7\rangle$

$$= 3\cdot 2 + -1\cdot 3 + 0\cdot 7$$

$$= 6 - 3$$

$$= \boxed{3}$$

c. $(2\mathrm{i}+2\mathrm{j}+4\mathrm{k})\times(-\mathrm{i}+\mathrm{j}-2\mathrm{k})$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 2 & 4 \\ -1 & 1 & -2 \end{vmatrix}$$

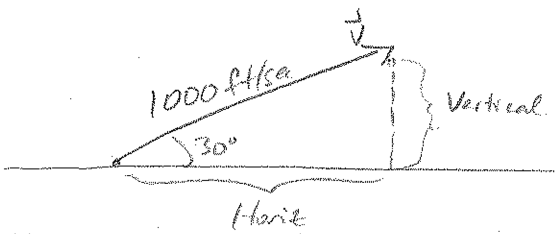
$$= \hat{\imath}\begin{vmatrix} 2 & 4 \\ 1 & -2 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & 4 \\ -1 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix}$$

$$= (-4-4)\hat{\imath} - (-4 - -4)\hat{\jmath} + (2 - -2)\hat{k}$$

$$= \boxed{-8\hat{\imath} + 4\hat{k}}$$

$$= \langle -8, 0, 4 \rangle$$

2. (7 points) A gun on the ground is fired at a speed of 1000 ft/sec at an angle of 30° above the ground. Find the horizontal and vertical components of the initial velocity.



Horiz Magnitude $= 1000 \cos 30°$ ft/sec

$= 1000 \sqrt{3}/2$

$\boxed{= 500\sqrt{3} \text{ ft/sec}}$ ✓

Vertical Magnitude $= 1000 \sin 30°$

$= 1000 \cdot 1/2$

$\boxed{= 500 \text{ ft/sec}}$

3. (8 points) Find the area of the triangle with vertices $\underset{P}{(1,0,0)}$, $\underset{Q}{(0,2,0)}$, and $\underset{R}{(0,0,3)}$.

$A = \frac{1}{2} |\vec{PQ} \times \vec{PR}|$

$\vec{PQ} = \langle -1, 2, 0 \rangle$

$\vec{PR} = \langle -1, 0, 3 \rangle$

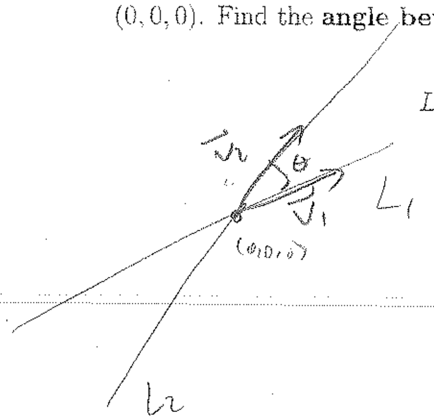
$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = \langle 6, 3, 2 \rangle$$

$$|\vec{PQ} \times \vec{PR}| = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{49} = 7$$

$$A = \frac{1}{2} |\vec{PQ} \times \vec{PR}| = \boxed{7/2}$$

4. (10 points) Below are parameterized equations for the two lines L_1 and L_2 which intersect at the origin $(0,0,0)$. Find the **angle between the lines** at the point of intersection.

$$L_1 : \begin{cases} x(t) = t \\ y(t) = 0 \\ z(t) = -\sqrt{3}t \end{cases} \qquad L_2 = \begin{cases} x(t) = t \\ y(t) = 2t \\ z(t) = -\sqrt{3}t \end{cases}$$



$$\vec{v}_1 = \langle 1, 0, -\sqrt{3} \rangle$$

$$\vec{v}_2 = \langle 1, 2, -\sqrt{3} \rangle$$

$$\cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1|\,|\vec{v}_2|} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\vec{v}_1 \cdot \vec{v}_2 = \langle 1, 0, -\sqrt{3} \rangle \cdot \langle 1, 2, -\sqrt{3} \rangle = 1 + 3 = 4$$

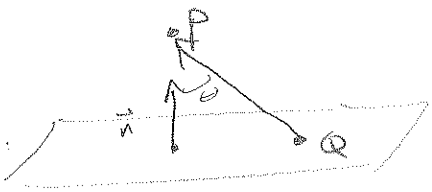
$$|\vec{v}_1| = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$$

$$|\vec{v}_2| = \sqrt{1^2 + 2^2 + 3} = \sqrt{8} = 2\sqrt{2}$$

$$\Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi/4}$$

5. (10 points) Find the distance between the point $(3, 6, 0)$ and the plane $2x - 2y + z = 6$.

P



$$\text{Distance} = \left|\text{Proj}_{\vec{n}} \overrightarrow{QP}\right| = \frac{|\overrightarrow{QP} \cdot \vec{n}|}{|\vec{n}|}$$

$P = (3, 6, 0)$, $Q = (0, 0, 6)$ ← Choose this

$\overrightarrow{QP} = \langle 3, 6, -6 \rangle$

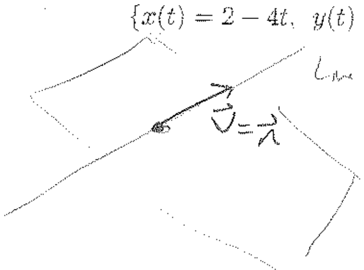
$\vec{n} = \langle 2, -2, 1 \rangle$

$\overrightarrow{QP} \cdot \vec{n} = \langle 3, 6, -6 \rangle \cdot \langle 2, -2, 1 \rangle = 6 - 12 - 6 = -12$

$|\vec{n}| = \sqrt{2^2 + 2^2 + 1} = \sqrt{9} = 3$

$$\boxed{\text{Dist} = \frac{|\overrightarrow{QP} \cdot \vec{n}|}{|\vec{n}|} = \frac{12}{3} = 4}$$

6. (10 points) Write an equation for the plane perpendicular to the line parameterized by $\{x(t) = 2 - 4t,\ y(t) = 3 + \frac{1}{2}t,\ z(t) = -1 + t\}$ and containing the point $(2, 3, -1)$.



Key Direction of line $(\vec{v})$

$=$ Normal vector of plane $(\vec{n})$

$\Rightarrow \vec{v} = \vec{n}$

$\vec{v} = \langle -4, \frac{1}{2}, 1 \rangle = \vec{n}$

Point $= (2, 3, -1)$

Plane (w/ $\vec{n} = \langle A, B, C \rangle$, Pt (x_0, y_0, z_0)) $\quad A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$$\Rightarrow \boxed{-4(x-2) + \frac{1}{2}(y-3) + (z+1) = 0}$$

7. (13 points) Write a parameterization for the line formed by the intersection of the following two planes:

$$\underbrace{x - 2y + z = 2}_{\text{Plane 1}}, \quad \underbrace{2x + y - 2z = 5}_{\text{Plane 2}}$$

$\vec{n}_1 = \langle 1, -2, 1 \rangle$

$\vec{n}_2 = \langle 2, 1, -2 \rangle$

Direction of line $= \vec{v} = \vec{n}_1 \times \vec{n}_2$ (perp to both normal vectors)

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 2 & 1 & -2 \end{vmatrix} = \langle 4-1, -(-2-2), 1+4 \rangle$$

$$= \langle 3, 4, 5 \rangle$$

Point on line

$$\begin{cases} x - 2y + z = 2 \\ 2x + y - 2z = 5 \end{cases} \xrightarrow{\text{Set}} \text{one variable to } 0 \text{ (e.g } x=0)$$

$$x=0 \Rightarrow \begin{cases} ① \; -2y + z = 2 \\ ② \; y - 2z = 5 \end{cases} \Rightarrow y = 5 + 2z$$

Plug in ② to ①

$$-2(5+2z) + z = 2$$

$$-10 - 4z + z = 2$$

$$-3z = 12$$

$$z = -4 \quad \Rightarrow y = 5 + 2(-4) = -3$$

Point $(0, -3, -4)$

(other easy points to find are $(9/4, 0, -1/4)$ and $(12/5, 1/5, 0)$)

$$\Rightarrow \boxed{\text{Line} \begin{cases} x(t) = 3t \\ y(t) = -3 + 4t \\ z(t) = -4 + 5t \end{cases}}$$

8. (20 points) Match the following graphs and equations:

- $x^2 + \frac{y^2}{4} = z + 1$

$z = 0 \Rightarrow$ Ellipse

$x = 0 \Rightarrow$ Parabola (conc up) $\Rightarrow$ Paraboloid (f)

$y = 0 \Rightarrow$ Parabola (conc up)

- $9x^2 + y^2 + z^2 = 9$

$z = 0$

$x = 0 \Rightarrow$ Ellipse $\Rightarrow$ Ellipsoid (e)

$y = 0$

- $z = y^2$

Indep of $x \Rightarrow$ "cylinder"

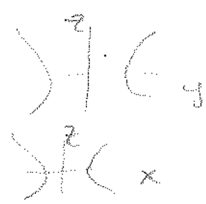
$x = 0 \Rightarrow z = y^2$ Parabola (c)

- $\frac{x^2}{4} + y^2 - z^2 = 1$

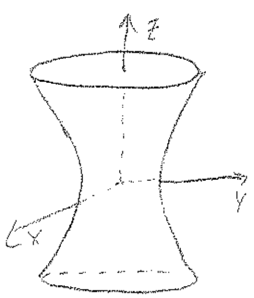
$z = 0 \Rightarrow$ Ellipse

$x = 0 \Rightarrow$ Hyperbola

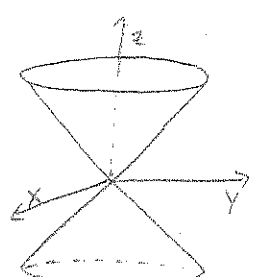
$y = 0 \Rightarrow$ Hyperbola



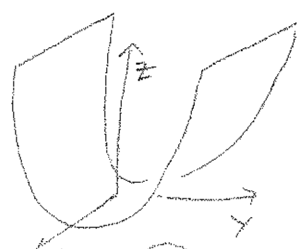
$\Rightarrow$ 1-sheeted hyperboloid (a)



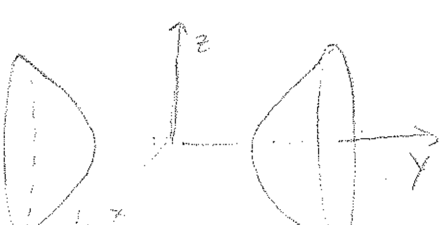
(a)



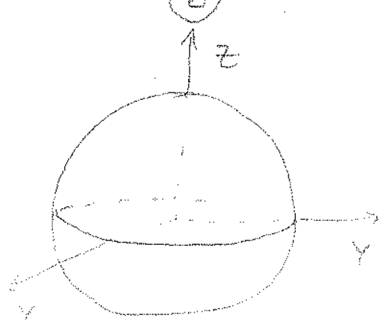
(b)



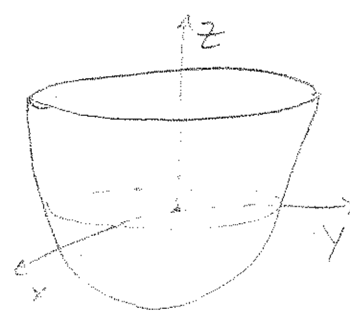
(c)



d



e



f

9. (7 points) Draw a graph of the equation $x^2 - y + z^2 = 0$.

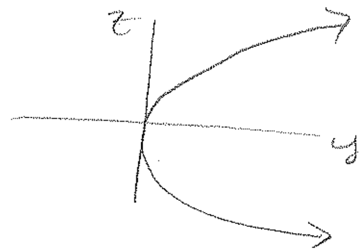
$$x^2 + z^2 = y$$

$y = 0 \Rightarrow$ Point

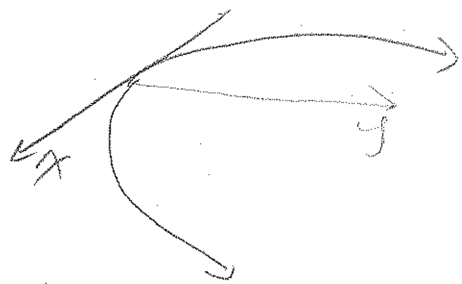
$y = \text{const} \Rightarrow x^2 + z^2 = y \Rightarrow$ Circle, $y > 0$

Nothing $y < 0$

$x = 0 \Rightarrow y = z^2$ Parabola



$z = 0 \Rightarrow \cancel{x}y = x^2$ Parabola



$\Rightarrow$ Paraboloid "going out in y-direction"