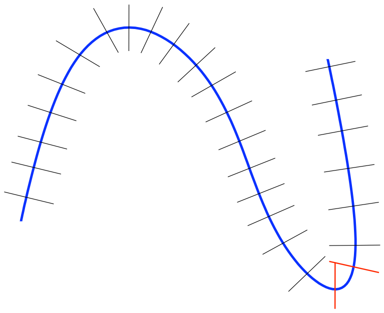
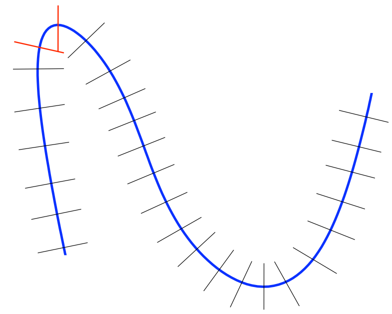


M421 HW 6

Due Wed. Dec 7

Eat the Hairy Worm



From Wade

Section	Page Number	Problems
11.4	414-415	2, 4, 9
11.5	422-424	6, 8, 9
11.6	433	3

Non-book Exercises

1) Let Γ be a curve in $\mathbf{R}^3$,

$$\Gamma = \left\{ \vec{g}(t) = (\gamma_1(t), \gamma_2(t), \gamma_3(t)) \,\middle|\, t \in [0,1] \right\},$$

where $\vec{g} \in \mathcal{C}^2([0,1], \mathbf{R}^3)$ satisfies $\|\vec{g}'(t)\| = 1$ for all $t \in [0,1]$. Suppose that $\vec{\phi}$ and $\vec{\psi}$ are $\mathcal{C}^1([0,1], \mathbf{R}^3)$ functions which satisfy $\|\vec{\phi}(t)\| = \|\vec{\psi}(t)\| = 1$.

Find a condition on $\vec{\phi}$ and $\vec{\psi}$ such that the map $F : \mathbf{R}^3 \mapsto \mathbf{R}^3$ given by

$$F(t, s_1, s_2) = \vec{g}(t) + s_1\vec{\phi}(t) + s_2\vec{\psi}(t),$$

defines a $\mathcal{C}^1$ coordinate system local to the curve γ. That is, find conditions which make F invertible in some neighborhood of each point of Γ with $F^{-1} \in \mathcal{C}^1$.

2) Let Γ be a smooth two-dimensional submanifold of $\mathbf{R}^3$, ie.

$$\Gamma = \left\{ \vec{g}(\vec{t}) = \left(\gamma_1(\vec{t}), \gamma_2(\vec{t}), \gamma_3(\vec{t})\right) \,\middle|\, \vec{t} = (t_1, t_2) \in [0,1] \times [0,1] \right\},$$

where $\vec{g} \in \mathcal{C}^2([0,1] \times [0,1], \mathbf{R}^3)$, satisfies $\left\| \frac{\partial \vec{g}}{\partial t_1}(\vec{t}) \times \frac{\partial \vec{g}}{\partial t_2}(\vec{t}) \right\| = 1$. Let $\vec{\nu}(\vec{t})$ be a normal to Γ at $\vec{g}(t)$ choosen to be locally smooth (there are two normals at each point, choose ν consistently). Show that the map

$$F(t, s) = \vec{g}(t) + s\vec{\nu}(t)$$

taking $\mathbf{R}^3$ to $\mathbf{R}^3$ is locally invertible in a neigborhood of each point $\vec{g}(\vec{t}) \in \Gamma$ with a C^1 inverse.