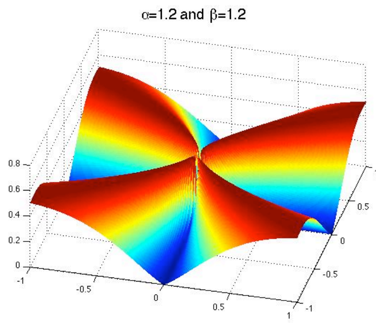
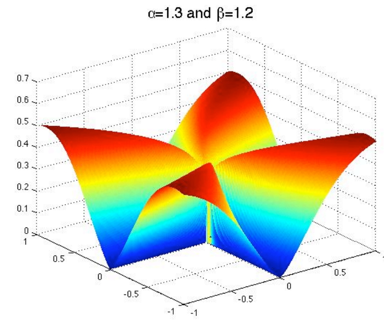


M421 HW 4

Due Fri. Oct 28



From Wade

Section	Page Number	Problems
9.3	319	2, 3
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Non-book Exercises

1) For each $\alpha > 0$ define the function $f_\alpha : \mathbf{R}^2 \mapsto \mathbf{R}$ by

$$f_\alpha(x, y) = \frac{x|y|^\alpha}{|x| + y^2}.$$

(a) For each $\alpha > 0$ determine if

$$\lim_{(x,y)\to(0,0)} f_\alpha(x, y) = 0.$$

(b) **Honor's Problem** For each $\alpha > 0$ determine the value of $\limsup_{(x,y)\to(0,0)} f_\alpha(x, y)$.

2) Consider $R^\infty = \{\vec{x} = (x_1, x_2, x_3, \ldots) \big| x_i \in \mathbf{R}, i = 1, 2, 3, \ldots\}$. The l^2-norm on R^∞ is

$$\|\vec{x}\|_2 = \left(\sum_{i=1}^{\infty} |x_i|^2\right)^{\frac{1}{2}}.$$

The space $l^2 = \{\vec{x} \in R^\infty \big| \|\vec{x}\|_2 < \infty\}$, is infinite dimensional. Show that the l^2 unit sphere, $S = \{\vec{x} \in l^2 \big| \|\vec{x}\|_2 = 1\}$, is closed, bounded, and **not** sequentially compact. That is, find a sequence from S which has no convergent subsequence.