M421 HW 2

Due Monday Oct. 3

From Wade

Section	Page Number	Problems
5.3	162-163	6, 7, 9
9.1	307	3, 4

Non-book Exercises

For the following exercises, the l^p norm is defined on $\mathbf{R}^n$ by

$$\|\vec{x}\|_p \equiv \left(\sum_{k=1}^{n} |x(k)|^p \right)^{\frac{1}{p}}.$$

1) Prove for $n = 2$ that $\frac{1}{\sqrt{2}}\|\vec{x}\|_1 \leq \|\vec{x}\|_2$.

2) Honors Problem For this problem let $p, q > 1$ satisfy $\frac{1}{p} + \frac{1}{q} = 1$. Such p and q are called conjugate exponents.

(a) Prove Young's inequality:

$$|ab| \leq \frac{|a|^p}{p} + \frac{|b|^q}{q},$$

for all conjugate exponents p, q and all $a, b \in \mathbf{R}$.

Hint: Compare the area of the box bounded by the lines $x = 0$, $y = 0$, $x = a$, and $y = b$ to the area between the x axis and the curve $y = x^{p-1}$ and between the y axis and the curve $x = y^{q-1}$. Use the fact that p, q conjugate exponents implies $(p-1)(q-1) = 1$, so the two curves are the same.

(b) Use Young's inequality to prove Hölder's inequality:

$$\sum_{k=1}^{n} |x(k)y(k)| \leq \|\vec{x}\|_p \|\vec{y}\|_q,$$

for all conjugate exponents p and q.

(c) Use Hölder's inequality to prove that

$$\frac{1}{n^{1/q}}\|\vec{x}\|_1 \leq \|\vec{x}\|_p,$$

for all $\vec{x} \in \mathbf{R}^n$ and all conjugate exponents p and q.

(d) Use Hölder's inequality to prove the triangle inequality for the l^p norm, $p \geq 1$. That is

$$\|\vec{x} + \vec{y}\|_p \leq \|\vec{x}\|_p + \|\vec{y}\|_p,$$

for all $\vec{x}, \vec{y} \in \mathbf{R}^n$.