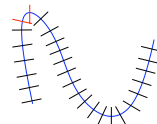


# M421 HW 1



Due Friday Sept. 23

From Wade

Section	Page Number	Problems
5.1	138-139	2b (note $P_n$ given in 2a), 3, 4, 5
5.2	151-152	5, 6, 8

## Non-book Exercises

**1)** Complete the proof of Remark 5.7. Show that if  $f : [a, b] \mapsto \mathbf{R}$  is bounded, and  $P, Q \in \mathcal{P}[a, b]$  satisfy  $Q \supseteq P$  then  $U(f, Q) \leq U(f, P)$ .

**2)** For  $n = 1, 2, \dots$ , define  $g_n(x) = 2xne^{-nx^2}$  for  $x \in [0, 1]$ .

(a) Show that  $\forall x \in [0, 1], \lim_{n \rightarrow \infty} g_n(x) = 0$ .

(b) Using the Fundamental Theorem of Calculus to evaluate the integrals on the left, show that

$$\lim_{n \rightarrow \infty} \int_0^1 g_n(x) dx \neq \int_0^1 \left( \lim_{n \rightarrow \infty} g_n(x) \right) dx.$$

**3)** Show that if  $f \in \mathcal{I}[a, b]$  then  $g(x) \equiv \sin(f(x)) \in \mathcal{I}[a, b]$ .