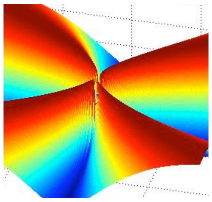
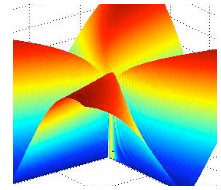


M254H HW 7

Due Friday March. 14



From Adams and Essex

Chapter	Page Number	Problems
12.9	740	7, 9, 12
10.7	610	23, 24, 28

Non-Book Problems

1) Let Γ be a curve in $\mathbf{R}^3$,

$$\Gamma = \left\{ \vec{\gamma}(t) = (\gamma_1(t), \gamma_2(t), \gamma_3(t)) \Big| \quad t \in [0,1] \right\},$$

where $\vec{\gamma} \in \mathcal{C}^2([0,1], \mathbf{R}^3)$ satisfies $\|\vec{\gamma}'(t)\| = 1$ for all $t \in [0,1]$. Suppose that $\vec{\phi}$ and $\vec{\psi}$ are $\mathcal{C}^1([0,1], \mathbf{R}^3)$ functions which satisfy $\|\vec{\phi}(t)\| = \|\vec{\psi}(t)\| = 1$.

Find conditions on $\vec{\phi}$ and $\vec{\psi}$ under which the map $F : \mathbf{R}^6 \mapsto \mathbf{R}^3$ given by

$$F(\vec{x}, t, s_1, s_2) := \vec{\gamma}(t) + s_1\vec{\phi}(t) + s_2\vec{\psi}(t) - \vec{x},$$

can be used to solve for $t = t(\vec{x})$, $s_1 = s_1(\vec{x})$, and $s_2 = s_2(\vec{x})$ such that $F(\vec{x}, t(\vec{x}), s_1(\vec{x}), s_2(\vec{x})) = 0$, for $\vec{x}$ close enough to Γ.

2) Let Γ be a smooth two-dimensional submanifold of $\mathbf{R}^3$, ie.

$$\Gamma = \left\{ \vec{\gamma}(\vec{t}) = \left(\gamma_1(\vec{t}), \gamma_2(\vec{t}), \gamma_3(\vec{t})\right) \Big| \quad \vec{t} = (t_1, t_2) \in [0,1] \times [0,1] \right\},$$

where the smooth function $\vec{\gamma}$ satisfies $\left| \frac{\partial \vec{\gamma}}{\partial t_1}(\vec{t}) \times \frac{\partial \vec{\gamma}}{\partial t_2}(\vec{t}) \right| = 1$. Let $\vec{\nu}(\vec{t})$ be the normal to Γ at $\vec{\gamma}(t)$. Show that the map

$$F(\vec{x}, \vec{t}, s) := \vec{\gamma}(t) + s\vec{\nu}(t) - \vec{x}$$

has can be solved for $\vec{t} = \vec{t}(\vec{x})$ and $s = s(\vec{x})$ such that $F(\vec{x}, \vec{t}(\vec{x}), s(\vec{x})) = 0$ for all $\vec{x}$ close enough to Γ.