Syllabus for the Analysis Qualifying Exam

Fall Semester 2003

Real Analysis. The primary reference is H. Royden, *Real Analysis, 3rd edition.* An additional reference is A.N. Kolmogorov and S.V. Fomin, *Introductory Real Analysis.*

1. Lebesgue outer measure on the set of real numbers, measurable sets.
2. Nonmeasurable sets.
3. Cantor set.
4. Simple functions, measurable functions, convergence properties of sequences of measurable functions, theorems of Egorov and Lusin.
5. Lebesgue integral on $\mathbb{R}$. Basic properties, convergence theorems.
6. The Lebesgue differentiation theorem. Absolutely continuous functions. Singular functions. The Cantor function.
7. Measures and integrals.
8. The product measure. The Fubini and Tonelli theorems.
9. Signed measures.
10. The Radon–Nikodym theorem.
11. The Hahn decomposition of signed measures.
12. The Stielties integral.
13. The Hölder and Minkowski inequalities. The L^p spaces. Completeness. Continuous linear functionals on L^p.

Complex Analysis. The primary reference is D. Sarason, *Notes on Complex Function Theory*, An additional reference is J. Bak and D.J. Newman, *Complex Analysis.*

1. Complex numbers, roots, stereographic projection, extended complex plane.
2. Complex differentiation, Cauchy–Riemann equations, holomorphic functions, conformal maps, harmonic functions.
3. Linear fractional transformations, fixed points, transitivity, properties and examples.
4. The exponential, trigonometric and hyperbolic functions. Arguments, logarithms, branches, roots.
5. Power series. Local uniform convergence, disk and radius of convergence, products of power series, differentiation of power series.

6. Complex integration.

7. Cauchy's theorem (for triangles, for convex regions), Cauchy integral formula for a circle. Infinite differentiability of holomorphic functions, Taylor series, Liouville's theorem, fundamental theorem of algebra, zeros of holomorphic functions, uniqueness theorem. Mean value theorem, maximum principle. The Schwarz inequality.

8. Laurent series and isolated singularities, classification of isolated singulariries, Casorati–Weierstrass theorem, residues.

9. Contours, winding numbers. Cauchy's theorem. Residue theorem. Meromorphic functions. The argument principle, Rouché's theorem. The local mapping theorem, inverses of holomorphic functions.

10. Evaluation of real integrals.

11. Simply connected domains. Winding number criterion. Existence of primitive, logarithms, harmonic conjugates. Normal families. The Stieltjes–Osgood theorem. The Riemann Conformal Mapping Theorem.

12. The Schwartz reflection principle.

13. Conformal maps from the unit disk onto itself, from the upper half-plane onto itself, from the upper half-plane onto the unit disk.

14. The Poisson intehral formula. The Dirichlet boundary value problem for the unit disk.