Math 930 Problem Set 7

Due Friday, November 7

Problem (7.1) Prove that if $\phi : G \to H$ is a Lie group homomorphism, then the differential $\phi_* : \mathfrak{g} \to \mathfrak{h}$ is a Lie algebra homomorphism.

Problem (7.2) Do Problem 8-14 on page 153 of the textbook.

Problem (7.3) Do Problem 8-7 on page 151 of the textbook.

Problem (7.4) As on page 38 of the textbook, one can define hyperbolic space $\mathbf{H}^n_R$ as the upper sheet ($t > 0$) of the two-sheeted hyperboloid

$$t^2 - \sum (x^i)^2 = R^2$$

in $\mathbb{R}^{n+1}$ with the Minkowski metric

$$m \;=\; dx^1 \otimes dx^1 + \cdots dx^n \otimes dx^n - dt \otimes dt.$$

Show that $\mathbf{H}^n_R$ has constant curvature $\dfrac{-1}{R^2}$ by following the two steps on Problem!8-9 on page 151 of the textbook.