# Topics in Gauge Theory

**Problem Set 4**

Due Friday, Nov 7

**Exercise 10.** Let $L^2(S^1)$ denote the Hilbert space of complex-valued functions on the unit circle $S^1 \subset \mathbb{C}$ with the hermitian inner product

$$\langle f, g \rangle \;=\; \int Re(f\overline{g}).$$

For each constant $c \in \mathbb{R}$, consider the 1-parameter family of operators

$$D_t = i\frac{d}{d\theta} + ct : L^2(S^1) \to L^2(S^1)$$

parameterized by $t \in \mathbb{R}$.

(a) Show that $D_t$ is self-adjoint.

(b) For which values of $c$ and $t$ is $D_t$ invertible?

(c) Compute the spectral flow of $D_t$ for the path $0 \le t \le L$, defined by

$$SF(D_t) \;=\; P - N \;\in\; \mathbb{Z}$$

where $P$ (reap. $N$) is the number of eigenvalues, counted with multiplicity, crossing 0 from negative to positive (reps. positive to negative) in the family. *You will have to assume that $D_t$ is invertible at the endpoints, and your answer will depend on both $L$ and $c$.*

*(d) Do parts (a), (b) and (c) for the similar family of *real* operators

$$E_t(f) \;=\; i\frac{df}{d\theta} + ct\overline{f} : L^2(S^1) \to L^2(S^1)$$