Erratum for: A Morse Theory for Equivariant Yang-Mills

(1) page 347: In equation (4.2) the second term in the numerator should read $e^{-i(l+1)\theta}$. The next line should begin “Now fix an odd integer $l > 1$.

(2) page 346: The reference to (4.1) on line 10 should be to (2.4).

(3) page 345: The proof of formula (3.3) is correct only if the center of the group $G$ is trivial. There is also some confusion between lifts and equivalence classes of lifts. The argument for a general compact group $G$ is given below. I thank Paul Kirk for this correction.

We first show that $B^H$ is a union $A_i/G_i$ of connections invariant under extensions of the $H$ action to $P$. Let $C(G)$ denote the center of $G$, which is also the center of $G$. Let $K = I/\sim$ parameterize the actions of extensions of $H$ by $C(G)$ on $P$ up to equivalence. Thus the elements of $I$ are groups $H_i$ which fit into exact sequences:

$$1 \to C(G) \to H_i \to H \to 1$$

Together with a lift of the $H$ action on $M$ to an $H_i$ action on $P$. We consider $H_i$ equivalent to $H_j$ if there is an isomorphism $\alpha : H_i \to H_j$ and a gauge transformation $\gamma \in G$ commuting the two actions, so $\gamma h \gamma^{-1} = \alpha(h)$.

Given $i \in I$, let $A_i = \{ A \in \mathcal{A} | h \cdot A = A \forall h \in H_i \}$ and $G_i = \{ g \in G | ghg^{-1}h^{-1} \in C(G) \forall h \in H_i \}$. Then $G_i$ acts on $A_i$ since if $A \in A_i$, $g \in G_i$, and $h \in H_i$, then there is a $c \in C(G)$ so that $hgA = ghcA$, but $cA = A$ since $C(G)$ lies in the stabilizer of $A$.

Set $B_i = A_i/G_i$. It is easy to check that $B_i = B_j$ when $i$ and $j$ are equivalent lifts. Let $A_i^*$ denote $A_i \cap A^*$, the irreducible invariant connections, and $B_i^* = A_i^*/G_i$. Let $B^{H*} = B^H \cap B^*$.

**Lemma.** The natural map $B_i^* \to B^{H*}$ is an embedding and $B^{H*}$ is a disjoint union

$$B^{H*} = \bigcup_{i \in K} B_i^*$$

where the notation $i \in K$ means that we choose one $i$ from each coset of $K$.

**Proof.** If $[A] \in B^{H*}$, pick a lift $A \in A$. Let $\mathcal{H}_A = \{ h \in Aut(P) | \pi(h) \in H, hA = A \}$, where $\pi : Aut(P) \to Diff(M)$. Since the stabilizer of $A$ in $G$ is just $C(G)$ the sequence

$$1 \to C(G) \to \mathcal{H}_A \to H \to 1$$

is exact (the third map is onto because $[A] \in B^{H*}$). Now $\mathcal{H}_A$ is a subgroup of $Aut(P)$ and so it acts on $P$, extending the $H$ action on $M$. Thus $\mathcal{H}_A \in I$ and a different choice of representative for $[A]$ yields an equivalent extension. Thus $\cup B_i^*$ maps onto $B^{H*}$. The restriction to $B_i^*$ is 1-1 since if $g \in G$, $A, B \in A_i^*$ so that $gA = B$, then for each $h \in \mathcal{H}_i$, $hgA = hB = B = gA = ghA$ so that $[g, h]$ stabilizes $A$ and hence $g \in G_i$. Finally, if $A \in A_i^*$ and $B \in A_j^*$ are gauge equivalent, then it is easy to see that $i$ and $j$ are equivalent lifts.

**Remark:** Given an extension $H_i$ of the $H$ action on $M$ to $P$, we can extend the $H$ action on $B$ to and $H_i$ action since if $gA = B$ and $h \in H_i$, then $hg^{-1}h^{-1} \in G$ and $(hg^{-1}h^{-1})hB = hA$, so $[hA] = [hB]$. Therefore the irreducible connections (mod gauge equivalence) left invariant by $H$ are the union of invariant connections under various $C(G)$ extensions of the $H$ action to $P$. 