Erratum for: A Morse Theory for Equivariant Yang-Mills

(1) page 347: In equation (4.2) the second term in the numerator should read $e^{-i(l+1)\theta}$. The next line should begin "Now fix an odd integer $l > 1$..."

(2) page 346: The reference to (4.1) on line 10 should be to (2.4).

(3) page 345: The proof of formula (3.3) is correct only if the center of the group G is trivial. There is also some confusion between lifts and equivalence classes of lifts. The argument for a general compact group G is given below. I thank Paul Kirk for this correction.

We first show that $\mathcal{B}^H$ is a union $\mathcal{A}_i/\mathcal{G}_i$ of connections invariant under extensions of the H action to P. Let $C(G)$ denote the center of G, which is also the center of $\mathcal{G}$. Let $K = I/\sim$ parameterize the actions of extensions of H by $C(G)$ on P up to equivalence. Thus the elements of I are groups H_i which fit into exact sequences:

$$1 \to C(G) \to H_i \to H \to 1$$

together with a lift of the H action on M to an H_i action on P. We consider H_i equivalent to H_j if there is an isomorphism $\alpha : H_i \to H_j$ and a gauge transformation $\gamma \in \mathcal{G}$ commuting the two actions, so $\gamma h \gamma^{-1} = \alpha(h)$.

Given $i \in I$, let $\mathcal{A}_i = \{A \in \mathcal{A} | h \cdot A = A \ \forall h \in H_i\}$ and $\mathcal{G}_i = \{g \in \mathcal{G} | ghg^{-1}h^{-1} \in C(G) \ \forall h \in H_i\}$. Then $\mathcal{G}_i$ acts on $\mathcal{A}_i$ since if $A \in \mathcal{A}_i$, $g \in \mathcal{G}_i$, and $h \in H_i$, then there is a $c \in C(G)$ so that $hgA = ghcA$, but $cA = A$ since $C(G)$ lies in the stabilizer of A.

Set $\mathcal{B}_i = \mathcal{A}_i/\mathcal{G}_i$. It is easy to check that $\mathcal{B}_i = \mathcal{B}_j$ when i and j are equivalent lifts.

Let $\mathcal{A}_i^*$ denote $\mathcal{A}_i \cap \mathcal{A}^*$, the irreducible invariant connections, and $\mathcal{B}_i^* = \mathcal{A}_i^*/\mathcal{G}_i$. Let $\mathcal{B}^{H*} = \mathcal{B}^H \cap \mathcal{B}^*$.

Lemma. *The natural map $\mathcal{B}_i^* \to \mathcal{B}^{H*}$ is an embedding and $\mathcal{B}^{H*}$ is a disjoint union*

$$\mathcal{B}^{H*} = \bigcup_{i \in K} \mathcal{B}_i^*$$

where the notation $i \in K$ means that we choose one i from each coset of K.

Proof. If $[A] \in \mathcal{B}^{H*}$, pick a lift $A \in \mathcal{A}$. Let $\mathcal{H}_A = \{h \in Aut(P) | \pi(h) \in H, hA = A\}$, where $\pi : Aut(P) \to Diff(M)$. Since the stabilizer of A in $\mathcal{G}$ is just $C(G)$ the sequence

$$1 \to C(G) \to \mathcal{H}_A \to H \to 1$$

is exact (the third map is onto because $[A] \in \mathcal{B}^H$.) Now $\mathcal{H}_A$ is a subgroup of $Aut(P)$ and so it acts on P, extending the H action on M. Thus $H_A \in I$ and a different choice of representative for $[A]$ yields an equivlaent extension. Thus $\cup_i \mathcal{B}_i^*$ maps onto $\mathcal{B}^{H*}$. The restriction to $\mathcal{B}_i^*$ is 1-1 since if $g \in \mathcal{G}$, $A, B \in \mathcal{A}_i^*$ so that $gA = B$, then for each $h \in \mathcal{H}_i$, $hgA = hB = B = gA = ghA$ so that $[g, h]$ stabilizes A and hence $g \in \mathcal{G}_i$. Finally, if $A \in \mathcal{A}_i^*$ and $B \in A_j^*$ are gauge equivalent, then it is easy to see that i and j are equivalent lifts. $\square$

Remark: Given an extension H_i of the H action on M to P, we can extend the H action on $\mathcal{B}$ to and H_i action since if $gA = B$ and $h \in H_i$, then $hg^{-1}h^{-1} \in \mathcal{G}$ and $(hg^{-1}h^{-1})hB = hA$, so $[hA] = [hB]$. Therefore the irreducible connections (mod gauge equivalence) left invariant by H are the union of invariant connections under various $C(G)$ extensions of the H action to P.