Math 868 — Homework 6

Due Monday, Nov. 5

1. Calculate $d\omega$ for the following forms on $\mathbb{R}^3$.
   (a) $\omega = z^2\, dx \wedge dy + (z^2 + 2y)\, dx \wedge dz$
   (b) $13x\, dx + y^2\, dy + xyz\, dz$
   (c) $f\, dg$ where f and g are functions
   (d) $(x + 2y^3)\, (dz \wedge dx + \frac{1}{2}\, dy \wedge dx)$.

2. Do Problem 14-6 in Lee, page 375. (In part (c) $\iota : S^2 \to \mathbb{R}^3$ is the inclusion).

3. Do Problem 14-7 on the same page.

4. To any three functions $P, Q, R \in C^\infty(\mathbb{R}^3)$, we can associate three opbjects on $\mathbb{R}^3$:
   - The vector field $X = P\, \frac{\partial}{\partial x} + Q\, \frac{\partial}{\partial y} + R\, \frac{\partial}{\partial z}$.
   - The 1-form $\omega_X = Pdx + Qdy + Rdz$.
   - The 2-from $\eta_X = P\, dy \wedge dz + Q\, dz \wedge dx + R\, dx \wedge dz$.

   Using this correspondence, show that d is related to the classical operators of vector calculus as follows:
   (a) For each $f \in C^\infty(M)$, $df = \omega_{\text{grad } f}$.
   (b) $d\omega_X = \eta_{\text{curl } f}$.
   (c) $d\eta_X = (\text{div}\, X)\, dx \wedge dy \wedge dz$.

   Then show that the fact that $d^2 = 0$ implies
   (d) curl(grad $(f)) = 0$
   (e) div(curl $(X) = 0$