Math 868 — Homework 9

Due Monday, Nov 12

These problems relate to Chapter 12 in Lee's textbook. All vector spaces are real vector spaces.

1. Do Problem 12-6 in Lee, page 320. (In part (c) $\iota : S^2 \to \mathbf{R}^3$ is the inclusion).
2. Do Problem 12-7 on the same page.
3. In class we showed that $\mathcal{L}_X$ is a derivation and d and ι_X are both skew-derivations on $\Omega^* M$. Use these facts to and Lemma 31.2 from class to:
   (a) Prove that $d\iota_X + \iota_X d$ is a derivation.
   (b) Prove the Cartan formula $\mathcal{L}_X = d\iota_X + \iota_X d$. (This is easier than the proof in Lee page
4. Read Lee pages 314-319 on symplectic vector spaces and symplectic manifolds. Do Exercise 12.7 on page 314.
5. Prove that on a manifold M of dimension $2n$, a 2-form $\omega \in \Omega^2(M)$ is a symplectic form if and only if
   $$d\omega = 0 \qquad \text{and and each point } p \in M \qquad \underbrace{(\omega \wedge \cdots \wedge \omega)_p}_{n \text{ times}} \neq 0.$$
   Hint: You need only show that this last condition is equivalent to ω being non-degenerate at each point p. Thus this is actually a linear algebra problem, which is easy using standard form.
6. Do Problem 12-9 in Lee, page 321.
7. Do Problem 12-14 in Lee, page 322.