Math 868 — Homework 6

Due Monday, Oct 22

1. Do Problem 19-3 on page 515 of Lee (the words "space of smooth global sections of D" mean the space of all smooth vector fields that, at each point $p \in M$, lie in the distribution $D \subset T_pM$).

2. Do Problem 19-6 on page 516 of Lee (Lee defines 'flat chart" on page 500; it is the same as what I called the "canonical k-dimensional distribution on $\mathbf{R}^n$").

3. Read Lee pages 510-515, then do Problem 19-13 on page 517.

4. Read Lee pages 125-127, then answer this: Let V be a vector space with dual space V^*.
   (a) Define a map $\phi : V \to V^{**}$ that is natural (ie defined without using any basis).
   (b) Prove that ϕ is injective.
   (c) When V is finite-dimensional, prove that ϕ is an isomorphism.

5. Read Lee pages 127-134. Then prove Proposition 6.9a-e on pages 133-4.

6. Read Lee pages 134-136. Do Problem 6-5 on page 151.

7. Read Lee pages 136-138, then answer this: Let $\phi : R^3 \to \mathbf{R}^3$ be the map
$$\phi(x, y, z) = (u, v, w) = (xe^{yz}, x^3yz^2, z\sin x).$$
   and let ξ and η be the 1-forms
$$\xi = w\,dv + u^2\,dw$$
$$\eta = v^2\,du + u^3\,dv + dw.$$
   Compute the pullbacks $\phi^*\xi$ and $\phi^*\eta$.