

Math 868 — Homework 4

Due Monday, Oct. 8

1. Do Problem 7-1 on page 171 in Lee.
2. Do Problem 7-2 on the same page.
3. Do Problem 7-3 on the same page. For this, you can use the "Equidistribution Theorem" that says that when α is an irrational number then the set of all points $e^{2\pi i \alpha N}$ is dense in the unit circle.
4. A map $f : M \rightarrow N$ is called *proper* if inverse images of compact sets are compact.
 - (a) Invent a meaning for the phrase "a sequence $\{x_k\}$ converges to infinity" in a topological space X .
 - (b) Prove that if $f : X \rightarrow Y$ is proper and (with your definition) $x_k \rightarrow \infty$, then $f(x_k) \rightarrow \infty$.
 - (c) Give an example of a smooth map $f : \mathbf{R} \rightarrow \mathbf{R}$ that is not proper.
5. Do Problem 8-1 on page 201 in Lee.
6. Do Problem 8-2 on page 201 in Lee.
7. Do Problem 8-10 on page 202 in Lee (Don't just say "yes" or "no" — give a proof or counterexample!)
8. Do Problem 8-11 on page 202 in Lee.

Hint: Cover S with product charts $U_\alpha \times B_\alpha(\epsilon)$, extend to a cover of N by adding the open set $N \setminus S$, and take a partition of unity subordinate to this cover. Use this to extend the given function f .