Math 868 — Homework 3

Due Friday, Sept. 28

1. Suppose that vector fields X and Y are given in local coordinates $\{x^i\}$ by
$$X = \sum X^i \, \frac{\partial}{\partial x^i} \qquad \text{and} \qquad Y = \sum Y^j \, \frac{\partial}{\partial x^j}.$$
where the components X^i and Y^j are functions of the $\{x^i\}$. Write $[X, Y] = \sum Z^k \frac{\partial}{\partial x^k}$. Find a formula for the components Z^k in terms of the X^i and Y^j. (This is in the textbook, but find it on your own).

2. Do Problem 4.11 on page 101 in Lee.

3. Do Problem 4.12 on page 101 in Lee.

4. Do Problem 4.14 on page 102 in Lee. Then sketch the vector field Z in the xy plane.

5. Let X be a vector field with $X_p \neq 0$ at $p \in M$. Show that there exist local coordinates $\{x^1, \ldots, x^n\}$ with origin at p such that in a neighborhood U of p we have $X = \frac{\partial}{\partial x^1}$.

6. (a) Prove that a path-connected topological space is connected.
   (b) Prove that manifolds are locally path-connected.
   (c) Prove that a manifold M is connected if and only if it is path-connected.

7. Do Problem 18-1 on page 491 in Lee.

8. Do Problem 18-2 Parts (a) and (b) on page 491 in Lee. (Read the Example on page 473 first.)