Math 868 — HINT for Homework 11

To compute the cohomology of the genus g surface, USE THE FACT PROVED IN CLASS THAT for $M = S^2 \setminus \{k \text{ disjoint disks}\}$

$$\begin{cases} H^0(M) = \mathbf{R} \\ H^1(M) = \mathbf{R}^{2g} \\ H^2(M) = \mathbf{R}. \end{cases}$$

Here are two facts that are very useful in all Mayer-Vietoris arguments:

Lemma 0.1 *In the Mayer-Vietoris sequence for any decomposition $M = U \cup V$ in which $U \cap V$ is connected, the first connecting homomorphism is always zero, ie $\delta = 0$ in the sequence*

$$0 \longrightarrow H^0(M) \longrightarrow H^0(U) \oplus H^0(V) \longrightarrow H^0(U \cap V) \xrightarrow{\delta} H^1(M) \longrightarrow .$$

Proof. This is immediate from the definition of this δ: a closed 0-form $\omega \in \Omega^0(U \cap V)$ is a function f with $df = 0$, so $f = c$ is a constant. But then $\delta[\omega] = [d\beta_U \cdot \omega] = [d(c\beta_U)] = 0.$ □

Lemma 0.2 *If M is a compact connected n-dimensional manifold then $H^n(M) = \mathbf{R}$.*

Proof. To be done in class. □