Math 868 — Homework 10

Due Wednesday, Nov 21

These problems relate to Chapters 13 and 14 in Lee's textbook. All vector spaces are real vector spaces.

1. Prove that the rank of $\omega \in \Lambda^2 V$ is rank $\omega = 2\max\{k \,|\, \omega^k \neq 0\}$ where ω^k means $\underbrace{\omega \wedge \cdots \wedge \omega}_{k \text{ times}}$.
2. Read Lee pages 325-329. Do Problem 13-1 of page 346.
3. Read Lee pages 40 -42 on smooth covering maps (read through the statement of Proposition 2.12).
   (a) Copy down the definition of "smooth covering map".
   (b) Do Problem 13-2 on page 346.
4. Do Problem 13-3 on page 346.
5. Do Problem 13-5 on page 347.
6. (a) Show that any open subset of an orientable manifold is orientable and that the product of twp orientable manifolds is orientable.
   (b) Do Problem 14-1 on page 382.
7. Evaluate $\int_S x\,dy \wedge dz + y\,dx \wedge dy$ where S is the oriented surface parameterized by $\phi : [0,1] \times [0,1] \to \mathbf{R}^3$ by $\phi(u,v) = (u+v, u^2 - v^2, uv)$ and oriented by $dx \wedge dy$.
8. Let H be the upper hemisphere $\{(x,y,z) \mid x^2 + y^2 + z^2 = 1, z \geq 0\}$. Evaluate
$$\int_{\partial H} (x+y)\,dz + (y+z)\,dx + (x+z)\,dy$$
   directly and by Stokes' Theorem. (Use orientation form $dx \wedge dy$.)
9. Let R be a region in R^3 oriented by $dx \wedge dy \wedge dz$. Show that the volume of R is
$$\frac{1}{3}\int_{\partial R} z\,dx \wedge dy + x\,dy \wedge dz + y\,dz \wedge dx$$
   and use this to compute the volume of the ball B_R of radius R in $\mathbf{R}^3$.