Math 415

Homework Set 7

due Monday, March 12

All of the following problems are from Chapter 5 of the Sadun textbook.

1. Problem 5 of Section 5.5.

2. Problem 7 of Section 5.5.

3. Problems 1, 3 and 4 of Section 5.6. These three problems analyze the same Markov Process. Start by drawing a diagram like the one I drew in class, and write down the corresponding matrix.

For the next two problems, *you may find the eigenvalues and eigenvectors using the website in my email* (also link on class webpage). Note that:

- The "steady state" means the eigenvector v_1 with $\lambda = 1$, normalized so that the sum of its entries is 1.

- If A is a regular probability matrix, then $\mathbf{x}_n = cv_1 + s_n$ for some constant c and with $|s_n| \to 0$ as $n \to \infty$. The rate that $|s_n|$ shrinks can be estimated in two ways:

  (i) If A is an $m \times m$ matrix whose smallest entry is α, the proof of Theorem 5.1 in the textbook shows that $|s_n| \leq (1 - m\alpha)^n$.

  (ii) As shown in class, if λ_2 is the eigenvalue with the second-largest absolute value, then $|s_n| \leq \mu^n$ where $\mu = |\lambda_2/\lambda_1|$.

Note that (i) is quick because it doesn't require computing eigenvalues.

4. Problem 7 of Section 5.6.

5. Problem 8 of Section 5.6.

Midterm Exam The midterm exam will be given in class on Wednesday March 14. There will be no long computations, and no proofs (although I may ask you to give a short explanation of why something is true). For preparation,

1. Review all definitions; be sure that you can state definitions *precisely*. This usually requires memorizing definitions.

2. Review that portion of the textbook that we covered: Chapter 2 (except Section 2.5), Chapter 3 (Except Quotient spaces), Chapter 4, and Chapter 5 (except Section 5.7).

3. Also look through your notes because there are some topics that I did that are not in the textbook(e.g. The Expansion Theorem, the Reduction Theorem, and the Structure Theorem).