Math 415

Homework Set 6

due Monday, Feb 27

All of the following problems are from Chapter 5 of the Sadun textbook.

1. Problem 3 of Section 5.1.
2. Problem 4 of Section 5.1.

   Hint: Observe that $\det A = 0$, $Tr(A) = 7$, and that all columns and all rows of A add up to 5. Find the eigenvalues by applying Tricks 1, 2 and 5 from Section 4.6. Also notice that $(1, 1, 1)$ is an eigenvector.
3. Problem 8 of Section 5.1.
4. Problem 2 of Section 5.2.
5. Problem 3 of Section 5.2.
6. Problem 6 of Section 5.2.

Hints for Problem 8 of HW Set 5:

(1) Before trying to show parts (a) and (b), show that the operators L and $P_\lambda = L - \lambda Id$ satisfy these simple facts:

- $LP_\lambda = P_\lambda L$.
- $LP_\lambda^n = P_\lambda^n L$ for any whole number n.
- $P_\lambda P_\mu = P_\mu P_\lambda$ for any eigenvalues λ and μ of L.

(2) Start part (c) this way:

Suppose that $\{v_1, v_2, \ldots, v_k\}$ are linearly dependent. Then there is a sum $\sum a^i v_i = 0$ with at least one $a^i \neq 0$. Take the shortest such sum, that is, one with the fewest number of non-zero a^i. After renumbering, we may assume that $a_1 \neq 0$. Now apply powers of P_{λ_1}: $\cdots$ *keep going* $\cdots$