Math 415

Homework Set 1

due Friday, Jan 13

The first 5 problems are taken from Sadun's book (circled problems on the scan download) with minor notation and wording changes.

1. Show that $W = \{(x, y) \in \mathbb{R}^2 \mid x + y = 0\}$ is closed under addition and scalar multiplication, and hence is a subspace of $\mathbb{R}^2$.

2. Show that $S = \{(x, y) \in \mathbb{R}^2 \mid x + y = 1\}$ is not a vector space.

3. Show that $S = \{(x, y) \in \mathbb{R}^2 \mid xy = 0\}$ (which is the union of the two coordinate axes) is not a vector space.

4. Is the set S of all vectors (x, y, z, w) in $\mathbb{C}^4$ that satisfy $x+y = z-w$ and $x+2y+3z+4w = 0$ a vector space? Why or why not?

5. Show that $L^2(\mathbb{R})$ = the set of all functions $f : \mathbb{R} \to \mathbb{R}$ such that $\int_{-\infty}^{\infty} |f(x)|^2 \, dx < \infty$ is a vector space. *Hint:* Square the integrand and use the fact that $|ab| \leq |a| \cdot |b|$ for all real numbers a, b.

6. Find 3 qualitatively different examples of maps between vector spaces that are not linear transformations.

7. Let I be the indefinite integration on the space $C[0, 1]$ of all continuous functions defined on the interval $[0, 1]$. Thus $(If)(x) = \int_0^x f(t) \, dt$. Show that I is a linear operator.

8. Let $\mathbb{R}[x]$ be the vector space of all polynomials in x (of any degree) with real coefficients. Show that the operator $D = \frac{d^2}{dx^2} + 17\frac{d}{dx}$ is a linear transformation $D : \mathbb{R} \to \mathbb{R}$.

9. Show that the projection $P : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $P(x, y, z) = (x, y)$ is a linear transformation.

10. Prove that (a) the composition of two linear transformations is a linear transformation, and (b) The inverse of a linear transformation (if it exists) is a linear transformation.

11. Prove that the image of a linear transformation $T : V \to W$ is a subspace of W.