Math 309, §2 Name SOLUTIONS

Quiz 1

2 pts **1.** Fill in the blanks: If $\{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\}$ is an orthonormal basis of a vector space V with inner product $\langle\ ,\ \rangle$, then each vector $\mathbf{x} \in V$ has a unique expansion

$$\mathbf{x} = \sum_{i=1}^{n} a_i\,\mathbf{e}_i \qquad \text{where } a_i = \langle \underline{\mathbf{x}}, \underline{\mathbf{e}_i} \rangle.$$

11 pts **2.** Use the Gram-Schmidt process to find an orthonormal basis of the subspace S of $\mathbb{R}^4$ spanned by the vectors

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \qquad \mathbf{x}_2 = \begin{pmatrix} 2 \\ 2 \\ 1 \\ 1 \end{pmatrix} \qquad \mathbf{x}_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 4 \end{pmatrix}$$

Number your steps and show your work.

Step 1 $\quad e_1 = \dfrac{x_1}{\|x_1\|} = \dfrac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$

Step 2. $\quad y_2 = x_2 - \langle x_2, e_1 \rangle e_1$

$\langle x_2, e_1 \rangle = \begin{pmatrix} 2 \\ 2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \cdot \frac{1}{2} = \frac{4}{2} = 2$

$= \begin{pmatrix} 2 \\ 2 \\ 1 \\ 1 \end{pmatrix} - 2 \cdot \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}$

$e_2 = \dfrac{y_2}{\|y_2\|} = \dfrac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}$

Step 3 $\quad y_3 = x_3 - \langle x_3, e_1 \rangle e_1 - \langle x_3, e_2 \rangle e_2$

$\langle x_3, e_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 4 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} = 2$

$\langle x_3, e_2 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 4 \end{pmatrix} \cdot \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} = 0$

$= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 4 \end{pmatrix} - 2 \cdot \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} - 0$

$= \begin{pmatrix} -1 \\ -1 \\ 1 \\ 3 \end{pmatrix}$

$e_3 = \dfrac{y_3}{\|y_3\|} = \dfrac{1}{\sqrt{12}}\begin{pmatrix} -1 \\ -1 \\ 1 \\ 3 \end{pmatrix}$

ON basis:

$$\left\{ \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}, \frac{1}{2\sqrt{3}}\begin{pmatrix} -1 \\ -1 \\ 1 \\ 3 \end{pmatrix} \right\}.$$