Homework Sets 1 – 5

All problems given by number are from *Linear Algebra with Applications*, 8th or 9th edition, by Steven Leon.

Class date	**Section covered**	**HW assigned (Do the listed problems by the following class)**
Wednesday Aug 29	1.1	1ac, 2ac, 3ac, 4ac, 5c, 6e, 7, 9.
Friday Aug 31	1.2	1, 2, 3 & 4, 5ef, 6c, 7, 8 (Hint: put in row echelon form), and 15 (see Application 1, page 17).
Wednesday Sept 5	1.3	1aceg, 2acf, 4,5, 7, 8a. **Then do Supplemental Problems 1–4 below.**
Friday Sept 7	1.4	1a, 2–4, 9 (Multiply A by itself 3 times), 12 (calculate $A \cdot A^{-1}$), 13, 20 (multiply both sides by (I-A) and simplify), 21 (calculate RR^T).
Monday Sept 10	1.5	1, 2, 3a, 10aeg, and then 7 (first use row operations to transform A to the identity, then express your operations as multiplication by elementary matrices (right-to-left!).

Supplemental Problems.

1. For $A = \begin{pmatrix} 1 & 3 \\ 4 & 7 \\ 11 & 10 \end{pmatrix}$, write down $-2A$, A^T, and verify that $(A^T)^T = A$.

2. For $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, verify that (a) $2(AB) = (2A)B = A(2B)$, (b)$(AB)^T = B^T A^T$.

3.(a) Write down two 2×2 matrices A and B (pick entries at random). Compute AB and BA and show that $AB \neq BA$.

(b) Show that the "identity matrix $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ commutes with any 2×2 matrix, i.e. $IB = BI$ for any $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

4. Consider the linear system $A\mathbf{x} = \mathbf{b}$ where $A = \begin{pmatrix} 1 & 3 \\ 1 & -3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, and $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

(a) Write this as a system of 2 equations in 2 variables.

(b) Solve the linear system by writing down the augmented matrix, reducing to RRE form, and finding the solution vector $\mathbf{x}$.

Homework Sets 6 – 10

All problems given by number are from *Linear Algebra with Applications*, 8th or 9th edition, by Steven Leon.

Notation: the book writes column vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ horizontally as $(x, y, z)^T$ to save space.

Caution: For those using Leon's 8th edition, the Section 3.2 Problems 18, 19ab, 21, 23 and 24 below correspond to Problems 15, 16ab, 18, 20 and 21 in the 8th edition (no change needed for Sections 3.1 or 3.3).

Class Date	**Section covered**	**HW assigned**
Wednesday Sept 12	1.5	**Do Supplemental Problem 5 below.** Then do Section 1.5 Problems 10h, 11, 12abc, 23, and 24. *Suggestions:* for both 23 and 24a), use the fact that if $A \cong B$ then $B = E_k E_2 \cdots E_1 A$ for some elementary E_i. For 24b), use 24a) and that fact that A is invertible if and only if $A \cong I_n$.
Friday Sept 14	3.1	4 and 6. For (4), $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ matrices. *Use the algebraic properties listed in Theorem 1.4.1 on page 47.* Then start the "Vector Space Axioms HW" on the last page of the handout by giving complete proofs to Lemmas 6, 7, and 8.
Monday Sept 17		Finish the Vector Space Axioms HW.
	3.2	Then do problems 1bc, 2ac, 3adf, 5a, and 6a in Section 3.2.
Wednesday Sept 19	3.2	Prepare for exam – see the Review Sheet. The exam covers Sections 1.1–1.5, 3.1, and the part of 3.2 done on vector subspaces (but not the part on span).
Friday Sept 21	**Exam 1**	No Homework for the weekend. *If you want to get ahead,* do Problems 11bc, 12a, 18, 19ab, 23, and 24 in Section 3.2 (these will be assigned on Monday).

Supplemental Problem.

5. (a) Write $A = \begin{pmatrix} 3 & 1 \\ 9 & 5 \end{pmatrix}$ as a product of elementary matrices. (b) Write A^{-1} as a product of elementary matrices.

Homework Sets 11 – 16

Class date	**Section covered**	**HW assigned**
Monday Sept 24	3.2	11bc, 12a, 18, 19ab, 23, and 24
	3.3	1ac, 2ad, 4c, 6, 8ac, 13, 14, 19.
Wednesday Sept 26	3.3	**Supplemental Problem 6** below.
	3.4	2ad (refers to 2ad in Section 3.3), 3abc, 5abc, 7, 11, 14a. For 7, take $a = 1$ and $b = c = 0$ to get a vector $\mathbf{v}_1$, etc.
Friday Sept 28	3.4	4, 8, 14, 15. For 14, use covert to vectors in $\mathbb{R}^3$ by $a + bx + cx^2 \leftrightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.
Monday Oct 1	3.5	1–4, **Supplemental Problem 7** below. 5, 6, 9a, 10, **Supplemental Problem 8** below.
Wednesday Oct 3	3.6	1, 3, 4ad, 6, 9 (use Rank-Nullity Theorem), 12 (use Theorems 3.61 & 3.61 for (a), give a 2×2 counterexample for (b), 19 (assume that $\mathbf{y} = \mathbf{0}$ and show that this leads to a contradiction), 26 (follow proof of Theorem 3.63a from class).

Supplemental Problems.

6. (a) Are the columns of $\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$ linearly independent vectors in $V = \mathbb{R}^3$?

(b) Are the columns of $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 0 \end{pmatrix}$ linearly independent vectors in $\mathbb{R}^3$? *You can answer without doing any calculation.*

7. Let $E = \{\mathbf{e}_1, \mathbf{e}_2\}$ be the standard basis of $\mathbb{R}^2$, and $B = \{\mathbf{b}_1, \mathbf{b}_2\}$ be the basis $\mathbf{b}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\mathbf{b}_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

(a) Find the transition matrices U_{EB} and U_{BE}.

(b) Use your answer to (a) to find the coordinates $[\mathbf{w}]_B$ of $\mathbf{w} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ in the B-basis.

8. Let $E = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis of $\mathbb{R}^3$, and let $B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be the basis

$$\mathbf{b}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \qquad \mathbf{b}_2 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \qquad \mathbf{b}_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$$

(a) Find the transition matrices U_{EB} and U_{BE}.

(b) Use your answer to (a) to find the coordinates $[\mathbf{w}]_B$ of $\mathbf{w} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$ in the B-basis.

Homework Sets 17–25

All of the problems below have the same numbers in Leon's 8th edition.

Class date	**Section covered**	**HW assigned**
Friday Oct 5	2.1	2, 3ace, 4, 5, 9 (expand on that row or column), 10, **Supplemental Problem 9** below.
Monday Oct 8	2.2	1ac, 2a, 3ace, 4, 5, 6, 7.
Wednesday Oct 10	2.2	5, 6, 7, Section 3.3 Problem 9, Section 3.4 Problem 13.
Friday Oct 12	2.3	2a and 2e.
	4.1	1abc, 3, 5abc, 6ab, 12.
Monday Oct 15	4.1	14, 16, 17bc, 19a.
Wednesday Oct 17	4.2	1abc, 2ab, 3c. Supplemental Problems 1–4 on the *Section 4.2 handout.*
Friday Oct 19	2.3	2a, 2c and 2e (on Cramer's Rule).
	4.2	4b, 5, 6, and 14. Supplemental Problems 5–13 on the *Section 4.2 handout.*
Monday Oct 22	Review	Prepare for exam – see the Review Sheet. Exam 2 covers Sections 3.3 – 3.6, 2.1–2.3, 4.1, and 4.2.
Wednesday Oct 24	**Exam 2**	No homework

Supplemental Problem.

9. For the matrix $B = \begin{pmatrix} 1 & 3 & -1 \\ 2 & 0 & 5 \\ 1 & 4 & 3 \end{pmatrix}$, calculate $\det A$ by (a) expanding along the first column, and (b) expanding along the second row. Are the results equal?

Homework Sets 26 – 32

Friday Oct 26	4.3	3–5, 8, 12, 13, 14.
Monday Oct 29	5.1	2ab, 3ac, 5, 7
Wednesday Oct 31	5.1	17, 18, 20, and **Supplemental Problem 10.**
Friday Nov 2	5.2	2, 3, 4, 8. For 3a, let $T = \{\mathbf{v} \in \mathbb{R}^3 \,\big|\, \mathbf{x} \cdot \mathbf{v} = \mathbf{y} \cdot \mathbf{v} = \mathbf{0}\}$. Use the definition of null space to show $N(A) = T$, then show $T = S^{\perp}$. For 8, give a precise proof. This generalizes problem 3a.
Monday Nov 5	5.2	1ad, 9 11 12a.
Wednesday Nov 7	5.3	1ab, 3, 4a, 9, 11.
Friday Nov 9	5.3	5, 6, and **Supplemental Problems 11 and 12.**
	5.4	1, 4.

Supplemental Problems.

10. Five students took aptitude exams in English, mathematics and science. Their scores are shown in the table on the back of this page.

(a) Find the correlation coefficients r_{EM}, r_{ES} and r_{MS} between the three pairs of variables.

(b) Which ones are positively correlated? Which are negatively correlated?

Student	English (E)	Math (M)	Science (S)
S1	61	53	53
S2	63	73	78
S3	78	61	82
S4	65	84	96
S5	63	59	71
Mean	66	66	76

11. Find the equation of the circle that gives the best least squares circle fit to the points $(-1, -2), (0, 2.4), 1.1, -4)$, and $(2.4, -1.6)$.

Method: use the formula $(x-a)^2 + (y-b)^2 = R^2$ for circles, then find the best-fit values of a, b, R with a calculator.

12. Given n points $(x_1, y_1), \ldots, (x_n, y_n)$ in the plane, find the line $y = ax + b$ of best fit. That is, use the Least Squares Method to determine the coefficients a, b in terms of the vectors $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_n)$.